Oil Shocks & Informality

Oscar Avila-Montealegre - Oscar Jaulín-Méndez - Marlon Salazar

August 31, 2026

Abstract

We explore how oil-price shocks affect informality in a small open economy. The analysis proceeds in two steps. We first provide empirical evidence of the causal effect of oil-price shocks on informality, showing that a positive shock increases the informality rate. We then rationalize this evidence with a small open economy model featuring tradable and non-tradable sectors. Consistent with the data, informality concentrates in non-tradable services; accordingly, the model assumes that the tradable sector employs only formal labor while the non-tradable sector also employs informal workers. The tradable sector further comprises export and import sub-sectors. A positive terms of trade shock expands the former and contracts the latter; the resulting income effect raises demand for non-tradable goods, boosting demand for informal workers and raising the informality rate. Consistent with the empirical evidence.

JEL Classification: F41, F16, E26, E32, O17
Keywords: *Open Economy; Trade and Labor; Informal Economy; DSGE*

1 Motivation

The relationship between oil price fluctuations and business cycle dynamics occupies a central place in international macroeconomics, particularly for commodity-exporting small open economies. A change in global oil prices alters a country's terms of trade, driving fluctuations in real national income, consumption, and investment. However, while the aggregate transmission of commodity shocks to output and exchange rates is well documented in empirical studies, significantly less attention has been paid to their transmission to labor market frictions (specifically labor informality). This adjustment channel is especially relevant in the Colombian context, where mining exports constitute approximately 38.8% of total exports and informal employment accounts for 56.9% of the workforce.[1] Motivated by the importance of these features of the Colombian economy, this paper investigates the mechanisms through which global oil price shocks affect labor informality.

The theoretical connection between oil shocks and informality operates through sectoral reallocation and relative price adjustments. A positive oil price shock generates a domestic spending effect, raising the relative price of non-tradable goods and expanding non-tradable demand. Crucially, in Colombia, informal labor is not uniformly distributed across the economy; it is concentrated in the non-tradable sector (e.g., transport, hospitality, and construction). Consequently, the propagation of oil shocks to overall informality depends on whether the spending effect draws workers into formal employment or expands informal non-tradable activities to meet rising domestic demand.

To empirically quantify this transmission mechanism, we estimate an instrumental-variable Bayesian vector autoregression (IV-BVAR) for Colombia using monthly data spanning from 2005 to 2025. We document two key findings. First, a 10% increase in brent oil prices generates a rise in aggregate informal employment, peaking at 0.3 percentage points within the first year following the shock. Second, this aggregate expansion is driven by strong sectoral heterogeneities: the shock significantly expands activity in non-tradable sectors characterized by high informality (such as construction, transportation, and hospitality) while producing a considerably muted effect on manufacturing, a tradable sector with a lower reliance on informal labor.

To rationalize these empirical facts, we construct a multi-sector, two-agent Small Open Economy DSGE model featuring high- and low-skilled households alongside formal and informal

[1] These values are calculated as averages for the period 2021–2025 using data from the National Administrative Department of Statistics (DANE).

labor margins. Production is divided into three distinct sectors (exportable, importable, and non-tradable) that differ in factor intensity and exposure to foreign price shocks. Following a terms-of-trade shock, changes in the relative price of exportables trigger a sectoral reallocation of capital and labor. Crucially, following empirical consensus (Fernández and Meza, 2015; Horvath and Yang, 2022), the informal labor margin is modeled exclusively within the non-tradable sector, serving as the primary buffer for low-skilled workers facing formal labor market entry costs or financial constraints. The quantitative model closely replicates the BVAR dynamics, demonstrating how an oil windfall drives non-tradable expansion and absorbs low-skilled labor into informal employment.

Three features are crucial for matching the empirical dynamics. First, wage rigidities amplify adjustments in labor quantities. Second, GHH preferences eliminate the wealth effect on labor supply, allowing the model to replicate the observed increase in informality. Third, household heterogeneity is essential: under a representative household specification, informality decreases on impact instead of increasing.

2 Literature review

Oil price and terms-of-trade (ToT) shocks are recognized as important sources of economic fluctuations, with heterogeneous effects across countries depending on the nature of the shock and each economy's exposure to foreign prices(Kilian, 2009; Peersman and Van Robays, 2012; Mendoza, 1995; Kose, 2002; Broda, 2004; Schmitt-Grohé and Uribe, 2018). The two are closely connected: oil price movements account for a substantial share of ToT variation, and the origin of an oil shock shapes the resulting external adjustment (Backus and Crucini, 2000; Kilian et al., 2009; Bodenstein et al., 2011). Our paper relates to two strands of literature: the effects of oil price and ToT shocks on economic activity, and their implications for labor-market outcomes and informality. Both are particularly relevant for Colombia, where external developments drive important business-cycle fluctuations (Cárdenas-Hurtado et al., 2018; Gómez and Suárez, 2019) and the labor market is characterized by persistent and high informality (Perry, 2010).

Since Hamilton (1983), a large literature has emphasized that the macroeconomic consequences of oil price movements depend critically on the underlying source of the shock (Kilian, 2009), motivating alternative identification strategies (Kilian and Murphy, 2014; Caldara et al., 2019; Baumeister and Hamilton, 2019; Känzig, 2021; Gazzani et al., 2024; Qureshi and Ahmad, 2025). For Colombia, positive oil price shocks tend to increase GDP

(Quero-Virla, 2016; Francis and Restrepo-Ángel, 2018), though the evidence is asymmetric and uncertain regarding their magnitude (Jiménez, 2010; González and Hernández, 2016). The adverse effects of oil price declines are better established: the 2014–2015 collapse deteriorated the ToT and contracted economuc activity (Toro et al., 2015), raised the country risk premium in a multi-sector DSGE model (Hamann-Salcedo et al., 2015), and strained fiscal accounts (Ojeda-Joya et al., 2016). Oil price movements are also consistently associated with exchange-rate movements—appreciation during price increases and substantial depreciation during declines (Jiménez, 2010; Toro et al., 2015; Hamann-Salcedo et al., 2015; Francis and Restrepo-Ángel, 2018; Melo-Becerra et al., 2020)—a channel that distributes the effects heterogeneously across sectors (Goda and García, 2015; Otero, 2020).

The macroeconomic effects of ToT shocks have been studied since the Harberger–Laursen–Metzler hypothesis, formalized by Mendoza (1995) in a general equilibrium framework where ToT disturbances account for a large share of GDP variability; reduced-form SVAR estimates, however, are considerably smaller (Schmitt-Grohé and Uribe, 2018). Related work examines whether ToT shocks behave like productivity shocks (Kehoe and Ruhl, 2008) and how transmission depends on the exchange-rate regime (Broda, 2004; Edwards and Yeyati, 2005). For small open economies, commodity price booms raise output and investment—amplified because borrowing spreads move inversely with commodity prices—and contractions deepen when prices fall (Drechsel and Tenreyro, 2018). The effects also depend on the source of the shock: export price shocks generate the traditional response and account for a larger share of output fluctuations, while import price shocks produce more muted effects (Di Pace et al., 2025). For Colombia, ToT and commodity price shocks are important drivers of activity, investment, and the real exchange rate (Echavarría et al., 2012; Camacho and Perez-Quiros, 2014; Gómez and Suárez, 2019; Poncela et al., 2017), a link strengthened by the concentration of exports in oil and a small number of commodities (Hernandez Jimenez, 2013; Toro et al., 2015).

Oil price shocks affect job-market flows and employment, with impacts concentrated in oil-importing countries, oil-intensive sectors, and male workers, while employment gains in oil-exporting countries are more modest (Ordóñez et al., 2011; Gomes et al., 2025). ToT shocks induce labor reallocation across tradable and non-tradable sectors, with the magnitude depending on wage flexibility and labor-market distortions (Edwards, 1988; Chaudhuri and Biswas, 2018; Bodenstein et al., 2018; Naraidoo and Paez-Farrell, 2023). Evidence on informality remains limited and points to heterogeneous responses: informal self-employment can expand procyclically through relative demand shocks to the non-tradable sector (Fiess et al., 2010); oil price increases contract the informal transport sector (Chaudhuri and Kar, 2023); and ToT improvements reduce the informal employment rate, though the effect fades after the first year (Jacho et al., 2024). For Colombia, positive oil price shocks reduce unemployment (Quero-Virla, 2016), with heterogeneous sectoral responses (Jiménez, 2010; Otero, 2020). General-equilibrium models that explicitly incorporate the formal–informal margin find that a larger informal sector buffers aggregate fluctuations (Lambert et al., 2020), that commodity booms reallocate resources toward the formal non-tradable sector and reduce informal employment (Ballesteros, 2016), and that a price collapse produces the mirror result—rising informality and formal unemployment—while a sustained boom generates formalization (Arguello, 2017; Arguello et al., 2016).

3 Empirical Strategy

We assess the transmission of oil price shocks to informality using an Instrumental Variable Bayesian Vector Autoregression (IV-BVAR) framework. This methodology leverages the advantages of Bayesian VARs, which mitigate the curse of dimensionality inherent to multivariate systems by applying shrinkage priors Koop and Korobilis (2010). Structural identification is achieved via external instruments, following the methodology established by Stock and Watson (2012) and Mertens and Ravn (2013). Incorporating external proxies ensures identification of the structural impact vector, isolating exogenous oil price variation from endogenous macroeconomic dynamics.

Abstracting from deterministic components and exogenous controls for ease of exposition, consider the following reduced-form VAR representation:

$$Y_t = \beta(L)Y_t + u_t \tag{1}$$

where Y_t denotes an $(n \times 1)$ vector of endogenous macroeconomic variables, and $\beta(L) = \sum_{p=1}^{P} B_p L^p$ is a lag polynomial of order P. The term u_t represents an $(n \times 1)$ vector of reduced-form innovations, which map to the underlying structural shocks, ε_t, via a linear transformation:

$$u_t = A\varepsilon_t \tag{2}$$

Consequently, the variance-covariance matrix of the reduced-form innovations is given by:

$$\Sigma_u = E[u_t u_t'] = E[A\varepsilon_t \varepsilon_t' A'] = AA' \tag{3}$$

Conditional on the estimated reduced-form parameters $\hat{\beta}(L)$ and the structural impact matrix A, the structural impulse response functions (IRFs) are recovered as:

$$IRF = (I - \hat{\beta}(L))^{-1} A \tag{4}$$

Due to the symmetry of Σ_u, the structural impact matrix A cannot be uniquely identified from the reduced-form estimates alone. To resolve this indeterminacy and trace the dynamic effects of a specific structural innovation (namely, an unanticipated oil price shock) we employ a proxy variable approach. Incorporating an external instrument isolates the strictly exogenous variation in oil prices, purging the impact estimates of endogenous confounding factors.

Following Stock and Watson (2018), let Z_t denote a scalar instrument that satisfies the standard relevance condition ($E[\varepsilon_{1,t} Z_t] = \alpha \neq 0$) and the contemporaneous exogeneity condition ($E[\varepsilon_{2:n,t} Z_t] = 0$). For notational convenience, we index the oil price as the first variable in the system, so that the targeted structural oil price shock is denoted by $\varepsilon_{1,t}$. The covariance between the reduced-form errors and the instrument is:

$$E[u_t Z_t] = E[A\varepsilon_t Z_t] = AE\begin{bmatrix} \varepsilon_{1,t} Z_t \\ \varepsilon_{2:n,t} Z_t \end{bmatrix} = A\begin{bmatrix} \alpha \\ 0 \end{bmatrix} = \begin{bmatrix} A_{1,1}\alpha \\ A_{2:n,1}\alpha \end{bmatrix} \tag{5}$$

Imposing the standard unit-effect normalization (Stock and Watson (2018)), we set the contemporaneous impact of the structural shock on its own indicator to unity ($A_{1,1} = 1$). The remaining elements of the first column of the impact matrix are then identified via the ratio of the covariances:

$$\frac{E[u_{i,t} Z_t]}{E[u_{1,t} Z_t]} = A_{i,1} \tag{6}$$

The dynamic responses of the macroeconomic variables to the oil price shock therefore correspond to the first column of the IRF matrix:

$$IRF_{\bullet,1} = (I - \hat{\beta}(L))^{-1} A_{\bullet,1} \tag{7}$$

Empirically, Equation 6 implies that the elements of the structural impact vector $A_{\bullet,1}$ can be identified via a Two-Stage Least Squares (2SLS) regression. Specifically, we regress the reduced-form residuals of the remaining macroeconomic variables, $u_{i,t}$, on the oil price residual, $u_{1,t}$, instrumenting the latter with the external proxy Z_t.

The estimation relies on a standard Normal-Inverted-Wishart (NIW) conjugate prior under a Minnesota-type specification.[2] This conjugate prior structure yields closed-form posterior distributions, enabling exact parameter sampling while improving inference in moderate sample sizes. Finally, following Lenza and Primiceri (2022) we treat the COVID-19 period as an episode of unusually large macroeconomic shocks rather than as evidence of a structural change in the economy's transmission mechanism. Their approach models the pandemic through a temporary increase in shock variances, with volatility parameters that capture the exceptional observations observed during the pandemic lockdown. By explicitly accounting for this volatility surge, the estimation procedure effectively downweights pandemic observations when identifying VAR parameters, preventing a small number of extreme observations from distorting the estimated dynamics.

4 Data and Results

We estimate the BVAR model using monthly data spanning January 2005 to December 2025. Because the main objective of the analysis is to evaluate the transmission of oil price shocks to Colombia's informal labor market, the core variables of interest are the Brent crude oil price and the informality rate, measured as the share of informal workers in total employment. To control for broader macroeconomic dynamics, we additionally include the terms of trade and aggregate gross domestic product (GDP). All variables are expressed in natural logarithms, except for the informality rate, which is reported in percentage terms. Further details on data sources and variable construction are provided in Appendix A.1.

To identify structural oil price shocks, we employ the oil price news instrument developed by Känzig (2021), which captures changes in oil supply expectations driven by OPEC announcements. For these announcements to serve as a valid instrument, they must strictly satisfy the exclusion restriction; namely, they must not reveal new, confounding information regarding global oil demand, broader economic activity, or geopolitical developments.

[2]The hyperparameter vector governing prior tightness is calibrated with overall tightness $\lambda = 0.2$, constant tightness $\epsilon = 0.001$, and sum-of-coefficients tightness $\tau = 10\lambda$. Endogenous variables are specified in levels, with the prior mean of the first autoregressive lag set to unity ($\delta = 1$) to reflect persistent series.

Two factors alleviate this "information effect" concern. First, unlike central banks—which systematically respond to macroeconomic data—OPEC's decisions are heavily influenced by political considerations and are less systematically tied to real-time economic conditions. This is corroborated by financial press coverage, which overwhelmingly interprets OPEC meetings as signals about production quotas rather than demand fundamentals. Second, to fully purge any residual informational confounding, the instrument utilizes an informationally robust surprise series. Akin to the narrative orthogonalization employed by Romer and Romer (2004) in the monetary policy literature, this refined measure is constructed by removing the effects of revisions to OPEC's own global demand forecasts. This rigorous refinement ensures that the instrument isolates genuinely exogenous shifts in oil supply expectations, guaranteeing the robustness of our identification strategy.

Figure 1 displays the impulse responses of the baseline four-variable system to an oil supply news shock, normalized to raise the Brent price by 1 % on impact. The oil price response is consistent with the interpretation of the shock as news about future supply: rather than peaking on impact, the Brent price continues to build after the announcement, reaching a maximum of about 1.4 % after two months and remaining roughly 0.5 % above baseline four years later. Colombia's terms of trade respond immediately, improving by 0.4 % on impact and peaking at 0.5 % in the following month; thereafter, they revert only slowly, with credible bands that exclude zero over virtually the entire response horizon.

Aggregate output responds positively, rising by up to 0.03 % during the first year, consistent with the income effect of the terms-of-trade windfall, and the response moderates at longer horizons. The informality rate, by contrast, builds up gradually, peaking at approximately 0.03 percentage points after about one year, an increase that is precisely estimated over most of the forecast horizon. Taken together, the results establish the causal chain at the core of our analysis: an oil price shock improves Colombia's terms of trade, generates a positive income effect on domestic activity, and produces a gradual and persistent increase in the informality rate. The implied magnitudes are economically meaningful: a 10 % rise in the Brent price, which improves the terms of trade by about 5 %, raises labor informality by roughly 0.3 percentage points.

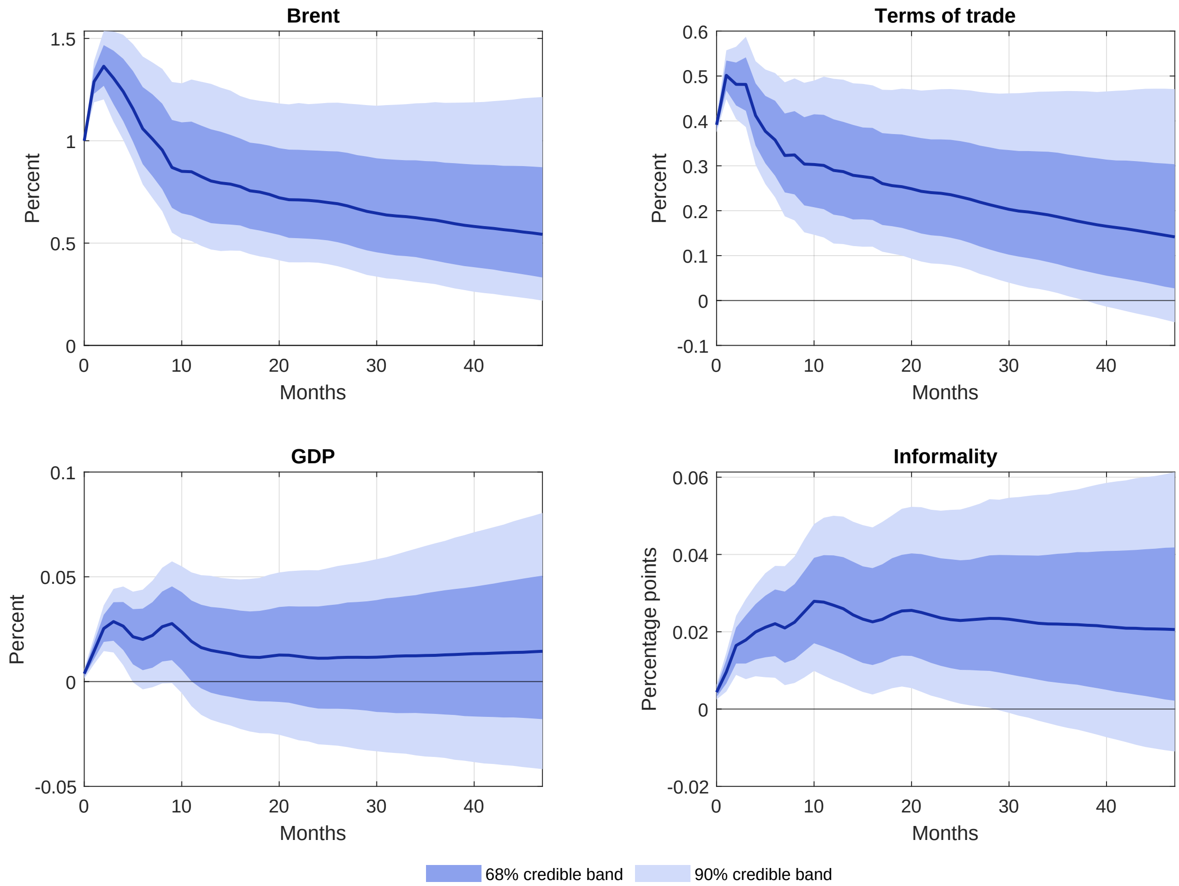

Figure 1: Impulse responses to an oil supply news shock: baseline model

Note: The figure reports impulse responses from the baseline four-variable BVAR, including the Brent price, the terms of trade, GDP, and the informality rate, estimated on monthly Colombian data for 2005M1–2025M12 with 12 lags and a Minnesota prior. The volatility of the pandemic period is controlled following Lenza and Primiceri (2022), with month-specific scale factors for March–December 2020 estimated by maximizing the marginal likelihood. The shock is identified using the oil supply news instrument of Känzig (2021) and normalized to raise the Brent price by 1 % on impact. Solid lines denote posterior medians; dark and light shaded areas denote 68 and 90 % pointwise credible bands, respectively. Responses of the Brent price, the terms of trade, and GDP are expressed in %; the response of the informality rate is expressed in percentage points. The horizon is in months.

4.1 Inspecting the Mechanism

We hypothesize that the response of aggregate labor informality to oil price shocks is driven by the asymmetric adjustment between the tradable and non-tradable sectors of the economy. Mechanically, an unanticipated increase in international oil prices improves the terms of trade (TOT) and leads to an appreciation of the real exchange rate (RER). This real appreciation erodes the external competitiveness of non-oil traditional export sectors—a classical

Dutch disease transmission channel. The resulting contraction in non-oil tradable activity, combined with an income effect that boosts domestic demand for non-tradable goods and services, reallocates activity toward non-tradable industries. Because non-tradable sectors exhibit a substantially higher baseline incidence of informal employment than formal tradable sectors, this structural reallocation ultimately elevates overall labor informality. To evaluate this mechanism empirical, we analyze the dynamic responses of four key economic sectors: manufacturing (tradable), construction (non-tradable), transport (non-tradable), and hospitality (non-tradable).

These sectors clearly illustrate the stark contrast in tradability across industries. Beyond mining and extraction, manufacturing constitutes Colombia's primary traditional export sector. According to official data from DANE, the manufacturing sector accounted for an average of 49.1% of total national exports between 2005 and 2025 (Figure 2). Agriculture represents the next largest non-mining export category at 7.0% over the same period, whereas Transport, Hospitality, and Construction toghether account for less than 1.0% of total exports, confirming their predominantly non-tradable nature.

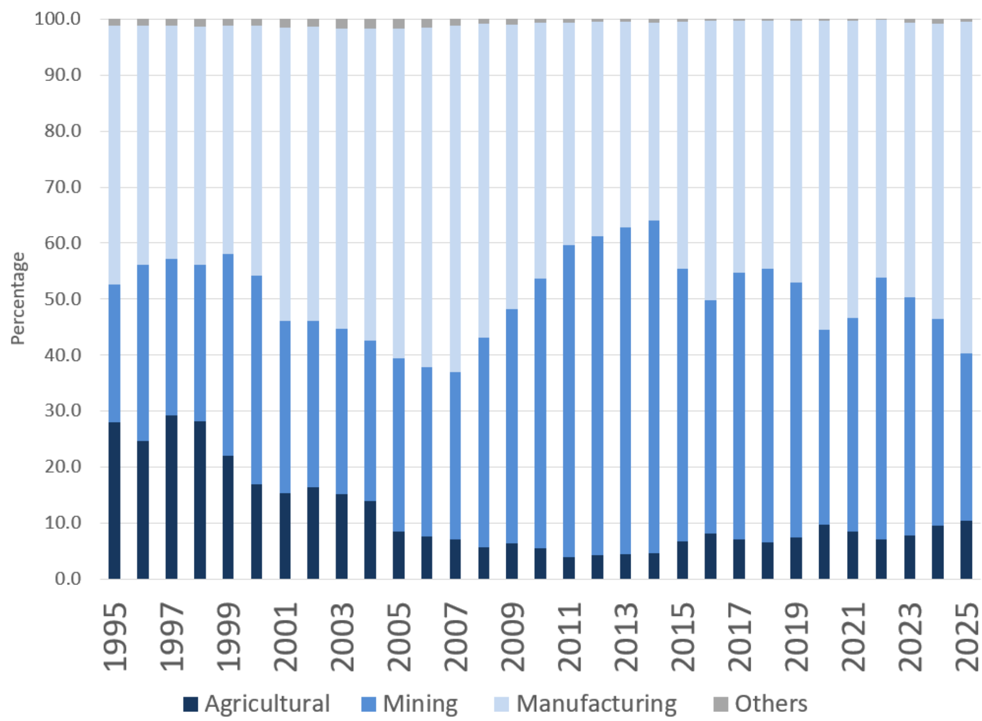

Figure 2: Share of Exports in Colombia
Source: DIAN-DANE, Total Exports, ISIC Rev.4 economic activity.

Crucially, these structural differences in tradability align directly with patterns of labor formalization. According to DANE, the average informality rate in manufacturing is 25.1 percentage points lower than the average informality rate observed in transport, hospitality, and construction combined. This confirms that informal labor is concentrated in non-tradable activities. Furthermore, while overall labor informality has exhibited a downward trend over time, the structural informality gap between these tradable and non-tradable sectors has remained highly persistent (Figure 3).

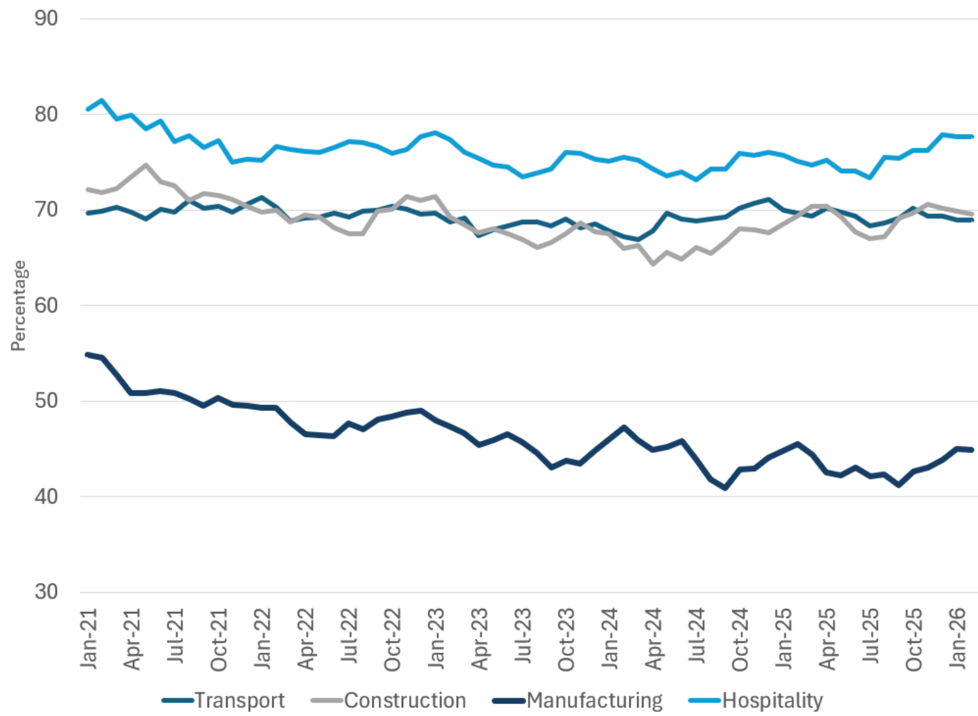

Figure 3: Share of Informal Workers by Sector
Source: DANE-GEIH (*Gran Encuesta Integrada de Hogares*). Total, formal, and informal employed population by ISIC Rev. 4 economic activity.

To empirically assess this reallocation mechanism, Figure 4 presents the structural impulse response functions of the extended nine-variable system to the oil supply news shock. Relative to the baseline specification, the model additionally includes the real exchange rate and four sectoral value-added measures. Manufacturing is included as a proxy for the tradable sector, whereas Construction, Transport, and Hospitality are included to capture the dynamics of the non-tradable sector. The responses of the Brent price and the terms of trade closely replicate those obtained in the baseline model, indicating that the identification remains robust despite the larger information set. The terms-of-trade improvement induced by the oil price shock operates through two complementary channels. First, the real exchange rate appreciates by approximately 0.2 %, with most of the adjustment occurring within the first year after the shock, consistent with the conventional Dutch disease mechanism. Second, the increase in national income generates an income effect that raises domestic demand, particularly in the non-tradable sector.

Consistent with this mechanism, activity expands more strongly in the non-tradable sectors. Construction increases by up to 0.09 % shortly after the shock, Transport peaks at around 0.07 % after approximately eight months, and Hospitality reaches a maximum increase of about 0.06 % around the fifth month following the shock. In contrast, Manufacturing exhibits only a modest increase that dissipates within the first year, as the appreciation-induced loss of external competitiveness gradually offsets the initial positive effect of the oil price shock. The relatively small increase in aggregate GDP, in line with the baseline specification, therefore reflects the net outcome of these two opposing forces

Given that the non-tradable sectors employ a substantially larger share of informal workers than Manufacturing (Figure 3), the post-shock reallocation of economic activity toward these sectors increases the demand for informal labor. As a result, the informality rate rises gradually, reaching a peak of approximately 0.03 percentage points around one year after the shock and remaining elevated thereafter. Taken together, these findings provide empirical support for the proposed transmission mechanism: improvements in the terms of trade increase aggregate informality through the combined effects of stronger demand for informality-intensive non-tradable activities and Dutch disease pressures that weaken the relative performance of the formal tradable sector.

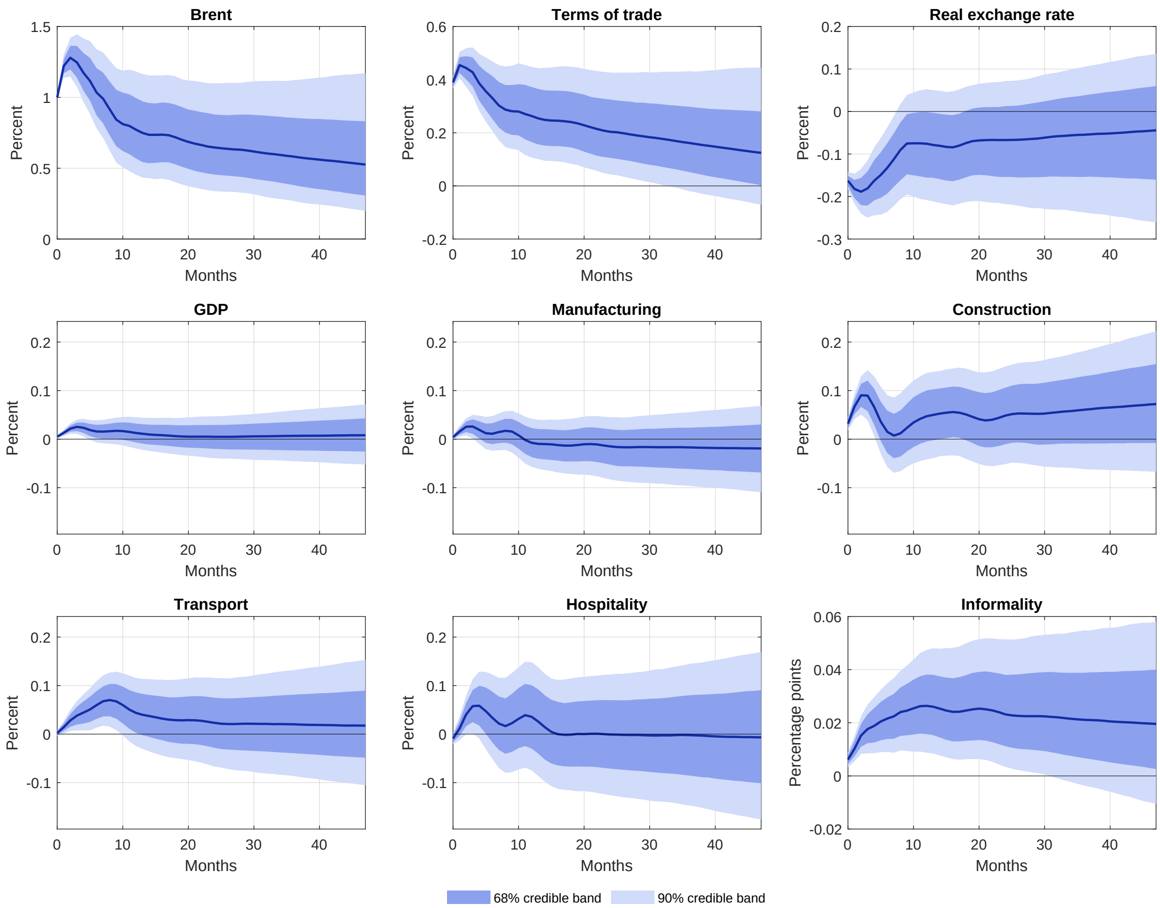

Figure 4: Impulse responses to an oil supply news shock: sectoral transmission

Note: The figure reports impulse responses from the extended nine-variable BVAR, which adds to the baseline specification the real exchange rate and sectoral value added in Manufacturing (tradable) and in Construction, Transport, and Hospitality (non-tradable), estimated on monthly Colombian data for 2005M1–2025M12 with 12 lags and a Minnesota prior. The volatility of the pandemic period is controlled following Lenza and Primiceri (2022), with month-specific scale factors for March–December 2020 estimated by maximizing the marginal likelihood. The shock is identified using the oil supply news instrument of Känzig (2021) and normalized to raise the Brent price by 1 % on impact. Solid lines denote posterior medians; dark and light shaded areas denote 68 and 90 percent pointwise credible bands, respectively. Responses of all variables are expressed in percent, except the informality rate, which is expressed in percentage points. GDP and the sectoral panels share a common vertical scale. The horizon is in months.

5 Model Structure

The empirical analysis presented in Section 4 establishes a close relationship between oil price shocks, terms-of-trade movements, and labor-market outcomes in Colombia. In particular, the results indicate that oil price shocks affect the terms of trade and, through their effects on domestic activity and the real exchange rate, have implications for formal and informal employment. The close comovement between oil prices and the terms of trade provides a natural basis for the theoretical analysis, as oil price movements account for a substantial share of terms-of-trade variation in commodity-exporting economies (Backus and Crucini, 2000; Toro et al., 2015). Since Colombia is a price taker in international oil markets, oil price fluctuations are determined independently of domestic economic conditions and constitute an external source of variation in the terms of trade. Accordingly, we model the external disturbance directly as a terms-of-trade shock, providing a parsimonious representation of the external price fluctuations considered in the empirical analysis and allowing us to study their transmission to the labor market.

The model follows (Schmitt-Grohé and Uribe, 2018) and describes a multi-sector, two-agent small open economy. It is designed to capture the joint dynamics of terms of trade fluctuations and labor informality. On the demand side, two types of households—high-skilled and low-skilled—differ in their access to financial markets and their labor supply (high and low productivity). On the production side, three sectors—exportable, importable, and non-tradable—differ in factor intensity and in their exposure to foreign price shocks. A terms-of-trade shock directly alters the relative price of exportables to importables, inducing sectoral reallocation of capital and labor. The informal labor margin is active exclusively in the non-tradable sector, consistent with the empirical consensus that informal employment is concentrated in non-tradable services (Fernández and Meza, 2015; Horvath and Yang, 2022). This margin represents the key margin of adjustment for low-skilled workers who cannot access—or choose not to access—formal employment in a given period.

5.1 Households

The economy is populated by a unit measure of households divided into a fraction N of high-skilled and a fraction $1-N$ of low-skilled. This distinction captures two empirically relevant features of emerging market economies: first, wealth concentration, whereby a minority of households holds the capital stock and has unrestricted access to international financial markets; second, labor market segmentation, whereby the majority of workers faces the choice between formal and informal employment. High-skilled households behave as Ricardian agents, smoothing consumption through capital accumulation and international borrowing. Low-skilled households, by contrast, hold no assets and finance consumption entirely from labor income and government transfers, making their consumption decisions tightly tied to current wage conditions in the formal and informal sectors.

5.1.1 High-skilled: N

The representative high-skilled household maximizes the following utility function. We consider GHH preferences, as in (Greenwood et al., 1988) to eliminate the wealth effect on labor supply.

$$E_0 \sum_{t+0}^{\infty} \beta^t \frac{1}{1-\sigma} \left(c_t^H - \psi_H \left(\frac{(h_t^{H,m})^{1+\eta_H}}{1+\eta_H} + \frac{(h_t^{H,x})^{1+\eta_H}}{1+\eta_H} + \frac{(h_t^{H,n})^{1+\eta_H}}{1+\eta_H} \right) \right)^{1-\sigma}$$

where c_t^H is high-skilled consumption and $h_t^{H,j}, j = m, x, n$ denotes hours worked in sector j. Hhs maximize their lifetime utility subject to the sequential budget constraint (normalized in units of final goods). The budget constraint equates high-skilled expenditure on consumption and sectoral investment to income from labor, capital rentals, and net foreign assets. Capital adjustment costs $\frac{\Phi_j}{2}(k_t^{H,j} - k_{t-1}^{H,j})^2$ slow the reallocation of capital across sectors, preventing the model from generating unrealistically sharp investment spikes in response to relative price changes.

The endogenous country interest rate premium, parameterized by Φ_B, ensures stationarity of the net foreign asset position—a standard device in small open economy models (Guajardo, 2008).

$$c_t^H + \sum_{j=m,x,n} \left(i_t^{H,j} + \frac{\Phi_j}{2}(k_t^{H,j} - k_{t-1}^{H,j})^2 \right) + p_t^\tau b_t^{H,*} = \tag{8}$$

$$p_t^\tau b_{t-1}^{H,*}(1 + r_{t-1}^*) + \sum_{j=m,x,n} \left(\tilde{w}_t^{H,j} h_t^{H,j} + R_t^{K,j} k_{t-1}^{H,j} \right)$$

$$k_t^{H,j} = (1-\delta) k_{t-1}^{H,j} + i_t^{H,j} \tag{9}$$

$$r_t^* = r_{ss}^* - \Phi_B \left(\exp(B_t^* - B_{ss}^*) - 1 \right) \tag{10}$$

where $i_t^{H,j}, k_t^{H,j}, \tilde{w}_t^{H,j}, R_t^{K,j}$ denote respectively: gross investment, the capital stock, the real wage, and the rental rate of capital in sector $j = m, x, n$. p_t^τ denotes the relative price of the tradable composite good in terms of final goods. From the first order conditions (FOCs) we have:

$$\left(c_t^H - \psi_H\left(\frac{(h_t^{H,m})^{1+\eta_H}}{1+\eta_H} + \frac{(h_t^{H,x})^{1+\eta_H}}{1+\eta_H} + \frac{(h_t^{H,n})^{1+\eta_H}}{1+\eta_H}\right)\right)^{-\sigma} = \lambda_t^H \tag{11}$$

$$\psi_H (h_t^{H,j})^{\eta_H} = \tilde{w}_t^{H,j} \tag{12}$$

$$p_t^\tau \lambda_t^H = \beta \mathbf{E}_t \lambda_{t+1}^H p_{t+1}^\tau (1 + r_t^*) \tag{13}$$

$$\lambda_t^H \left(1 + \Phi_j(k_t^{H,j} - k_{t-1}^{H,j})\right) = \beta \mathbf{E}_t \lambda_{t+1}^H \left(R_{t+1}^{H,j} + 1 - \delta + \Phi_j(k_{t+1}^{H,j} - k_t^{H,j})\right) \tag{14}$$

The first-order conditions are standard: the intratemporal labor supply conditions equates the marginal disutility of work to the after-tax real wage in each sector, while the Euler equations govern intertemporal consumption, investment, and foreign assets decisions.

5.1.2 Low-skilled: $1 - N$

On the other hand, the representative low-skilled household maximizes:

$$E_0 \sum_{t+0}^{\infty} \frac{\beta}{1-\sigma}\left(c_t^L - \psi_F\left(\frac{(h_t^{F,m})^{1+\eta_L}}{1+\eta_L} + \frac{(h_t^{F,x})^{1+\eta_L}}{1+\eta_L} + \frac{(h_t^{F,n})^{1+\eta_L}}{1+\eta_L}\right) - \psi_l\left(\frac{(h_t^{I,n})^{1+\eta_L}}{1+\eta_L}\right)\right)^{1-\sigma}$$

where c_t^L is low-skilled consumption and $h_t^{F,j}, j = m, x, n$ denotes formal hours worked in sector j, and $h_t^{I,n}$ informal hours in the non-tradable sector.

Hhs maximize their lifetime utility subject to the sequential budget constraint. Low-skilled households supply formal labor to all three production sectors and informal labor exclusively to the non-tradable sector. The utility specification distinguishes between the disutility of formal labor (ψ_F) and informal labor (ψ_I), reflecting differences in working conditions, degree of social protection, or psychic costs associated with each employment type. Because low-skilled households cannot save or borrow, their budget constraint equates consumption to the sum of formal and informal labor income across sectors, and lump-sum government transfers T_t.

$$c_t^L = \sum_{j=m,x,n} \tilde{w}_t^{F,j} h_t^{F,j} + \tilde{w}_t^{I,n} h_t^{I,n} + T_t \tag{15}$$

From the first order conditions (FOCs) we have that labor supply depends entirely on real wages:

$$\left(c_t^L - \psi_F \left(\frac{(h_t^{F,m})^{1+\eta_L}}{1+\eta_L} + \frac{(h_t^{F,x})^{1+\eta_L}}{1+\eta_L} + \frac{(h_t^{F,n})^{1+\eta_L}}{1+\eta_L} \right) - \psi_I \frac{(h_t^{I,n})^{1+\eta_L}}{1+\eta_L} \right)^{-\sigma} = \lambda_t^L \tag{16}$$

$$\psi_F (h_t^{F,j})^{\eta_L} = \tilde{w}_t^{F,j} \tag{17}$$

$$\psi_I (h_t^{I,j})^{\eta_L} = \tilde{w}_t^{I,n} \tag{18}$$

The wedge between formal wages $\tilde{w}_t^{F,j}$ and informal wages $\tilde{w}_t^{I,n}$ arises endogenously in equilibrium, driven by the payroll taxes that apply only to formal employment and by the relative disutilities of formal and informal work. When terms of trade improve, higher demand for exportable goods raises formal wages in that sector, which—through the common labor supply curve of low-skilled workers—puts upward pressure on formal wages economy-wide.

5.1.3 Aggregates

Aggregate consumption is the weighted average of high and low-skilled consumption $C_t = Nc_t^H + (1-N)c_t^L$. Since only high-skilled households have access to capital accumulation, investment decisions, and foreign bonds, the corresponding aggregates are $K_t^j = Nk_t^{H,j}$, $I_t^j = Ni_t^{H,j}$ for $j \in x, m, n$. Finally, aggregate labor is given by $H_t^{H,j} = Nh_t^{H,j}$ for $j \in x, m, n$ and $H_t^{I,n} = (1-N)h_t^{I,n}$.

5.2 Wage rigidities

The following problems are equivalent for all the types of labor (high and low-killed, formal and informal)

5.2.1 Labor packer

Competitive firms that aggregate heterogeneous labor into an aggregate input.

$$h_t^j = \left(\int_0^1 (h_t^j(l))^{\frac{\theta_j - 1}{\theta_j}} dl \right)^{\frac{\theta_j}{\theta_j - 1}} \tag{19}$$

Firms maximize profits:

$$\max w_t^j h_t^j - \int_0^1 w_t^j(l) h_t^j(l) dl$$

From the F.O.Cs we obtain the demand for a particular variety as a function of relative wages and aggregate labor of type j:

$$h_t^j(l) = \left(\frac{w_t^j}{w_t^j(l)} \right)^{\theta_j} h_t^j \tag{20}$$

5.2.2 Labor unions

Supply labor to firms at a price $w_t^j(l)$ and buy it from the households at a price $\tilde{w}_t^j$. They face wage rigidities a la Rotemberg, where the wage adjustment costs is in units of tradable goods is a quadratic frunction given by:

$$\Upsilon_t^j = \frac{\phi_j}{2} \left(\frac{w_t^j(l)}{w_{t-1}^j(l)} - 1 \right)^2 p_t^\tau q_t^\tau \tag{21}$$

The optimization problem is given by:

$$\max \Pi_t^j(l) = \left(w_t^j(l) - \tilde{w}_t^j \right) h_t^j(l) - \Upsilon_t^j$$

subject to:

$$h_t^j(l) = \left(\frac{w_t^j}{w_t^j(l)} \right)^{\theta_j} h_t^j$$

From the F.O.C:

$$\begin{aligned} \left((1-\theta_j)(w_t^j(l))^{-\theta} + \theta_j \tilde{w}_t^j (w_t^j(l))^{-\theta-1} \right) (w_t^j)^{\theta_j} h_t^j - \phi_j \left(\frac{w_t^j(l)}{w_{t-1}^j(l)} - 1 \right) \frac{1}{w_{t-1}^j(l)} p_t^\tau q_t^\tau \\ + \mathbf{E}_t \beta \frac{\lambda_{t+1}^H}{\lambda_t^H} \phi_j \left(\frac{w_{t+1}^j(l)}{w_t^j(l)} - 1 \right) \frac{w_{t+1}^j(l)}{(w_t^j(l))^2} p_{t+1}^\tau q_{t+1}^\tau = 0 \end{aligned}$$

Under the symmetric equilibrium, where $w_t^j(l) = w_t^j$ the wage Phillips curves are given by:

$$\left((1-\theta_j)+\theta_j\frac{\tilde{w}_t^j}{w_t^j}\right)h_t^j-\phi_j\left(\frac{w_t^j}{w_{t-1}^j}-1\right)\frac{1}{w_{t-1}^j}p_t^\tau q_t^\tau$$
$$+\mathbf{E}_t\beta\frac{\lambda_{t+1}^H}{\lambda_t^H}\phi_j\left(\frac{w_{t+1}^j}{w_t^j}-1\right)\frac{w_{t+1}^j}{(w_t^j)^2}p_{t+1}^\tau q_{t+1}^\tau=0$$

5.3 Firms

Production is organized through a set of competitive firms. In each sector, firms hire high-skilled and formal low-skilled labor from economy-wide competitive markets, and they rent capital from high-skilled households. A payroll tax τ^H on high-skilled labor and τ^F on formal low-skilled labor creates a wedge between the gross wage received by workers and the labor cost borne by firms. The non-tradable sector additionally hires informal low-skilled labor, which is exempt from payroll taxes. This tax differential is the key mechanism that sustains informal employment in equilibrium: it makes informal labor relatively cheaper for the firm, while low-skilled workers find informal work attractive when their disutility of informal work (ψ_I) is sufficiently low relative to the wage premium they would need to accept formal work. Government revenues from payroll taxes are rebated in full to low-skilled households as lump-sum transfers T_t (equation in the Macroeconomic Equilibrium block), so the model abstracts from fiscal policy questions and focuses on the allocative distortion introduced by the tax wedge.

5.3.1 Final Goods

Final consumption and investment goods are assembled by competitive aggregating firms using CES technology. The first tier combines a tradable composite q_t^τ and a non-tradable good q_t^n with elasticity of substitution θ_c and expenditure share ω_c on tradables. The resulting CES price index is normalized to unity, which defines the numeraire of the model.

$$C_t+\sum_{j=m,x,n}I_t^j=\left[\omega_c^{\frac{1}{\theta_c}}\left(q_t^\tau\right)^{\frac{\theta_c-1}{\theta_c}}+(1-\omega_c)^{\frac{1}{\theta_c}}\left(q_t^n\right)^{\frac{\theta_c-1}{\theta_c}}\right]^{\frac{\theta_c}{\theta_c-1}}$$

The optimization problem is given by

$$\max_{q_t^\tau, q_t^n} \left[\omega_c^{\frac{1}{\theta_c}} (q_t^\tau)^{\frac{\theta_c - 1}{\theta_c}} + (1 - \omega_c)^{\frac{1}{\theta_c}} (q_t^n)^{\frac{\theta_c - 1}{\theta_c}} \right]^{\frac{\theta_c}{\theta_c - 1}} - p_t^\tau q_t^\tau - p_t^n q_t^n$$

From the FOCs we have the relative demands and the optimal price level:

$$q_t^\tau = \omega_c (p_t^\tau)^{-\theta_c} \left(C_t + \sum_{j=m,x,n} I_t^j \right) \tag{22}$$

$$q_t^n = (1 - \omega_c)(p_t^n)^{-\theta_c} \left(C_t + \sum_{j=m,x,n} I_t^j \right) \tag{23}$$

$$1 = \left[\omega_c (p_t^\tau)^{1-\theta_c} + (1 - \omega_c)(p_t^n)^{1-\theta_c} \right]^{\frac{1}{1-\theta_c}} \tag{24}$$

5.3.2 Tradable goods

The second tier combines exportable goods q_t^x and importable goods q_t^m into the tradable composite with elasticity θ_τ and expenditure share ω_τ. This nested structure generates well-defined sectoral demand schedules that respond to changes in relative prices. A terms of trade improvement (p_t^x / p_t^m rising) shifts relative demand toward exportables within the tradable bundle, and shifts relative demand toward tradables in the final good bundle if p_t^τ rises relative to p_t^n.

$$q_t^\tau = \left[\omega_\tau^{\frac{1}{\theta_\tau}} (q_t^x)^{\frac{\theta_\tau - 1}{\theta_\tau}} + (1 - \omega_\tau)^{\frac{1}{\theta_\tau}} (q_t^m)^{\frac{\theta_\tau - 1}{\theta_\tau}} \right]^{\frac{\theta_\tau}{\theta_\tau - 1}}$$

The optimization problem is given by

$$\max_{q_t^x, q_t^m} p_t^\tau \left[\omega_\tau^{\frac{1}{\theta_\tau}} (q_t^x)^{\frac{\theta_\tau - 1}{\theta_\tau}} + (1 - \omega_\tau)^{\frac{1}{\theta_\tau}} (q_t^m)^{\frac{\theta_\tau - 1}{\theta_\tau}} \right]^{\frac{\theta_\tau}{\theta_\tau - 1}} - p_t^x q_t^x - p_t^m q_t^m$$

From the FOCs we have the relative demands and the optimal price level:

$$q_t^x = \omega_\tau \left(\frac{p_t^x}{p_t^\tau}\right)^{-\theta_\tau} q_t^\tau \tag{25}$$

$$q_t^m = (1-\omega_\tau) \left(\frac{p_t^m}{p_t^\tau}\right)^{-\theta_\tau} q_t^\tau \tag{26}$$

$$p_t^\tau = \left[\omega_\tau (p_t^x)^{1-\theta_\tau} + (1-\omega_\tau)(p_t^m)^{1-\theta_\tau}\right]^{\frac{1}{1-\theta_\tau}} \tag{27}$$

5.3.3 Exportable and importable goods

Exportable goods are produced using capital K_t^x, high-skilled labor $H_t^{H,x}$, and formal low-skilled labor $H_t^{F,x}$ via a Cobb-Douglas technology with sector-specific TFP A_t^x:

$$y_t^x = A_t^x (K_t^x)^{\alpha_x} (H_t^{H,x})^{\kappa_x} (H_t^{F,x})^{1-\alpha_x-\kappa_x} \tag{28}$$

Profit maximization yields three first-order conditions, one for each factor of production, which imply that the marginal product of each factor is equal to its real price.

$$\alpha_x \frac{p_t^x y_t^x}{K_t^x} = R_t^x \tag{29}$$

$$\kappa_x \frac{p_t^x y_t^x}{H_t^{H,x}} = (1+\tau^H)\tilde{w}_t^{H,x} \tag{30}$$

$$(1-\alpha_x-\kappa_x) \frac{p_t^x y_t^x}{H_t^{F,x}} = (1+\tau^F)\tilde{w}_t^{F,x} \tag{31}$$

Output in excess of domestic demand is exported. Net exports in value terms are therefore:

$$x_t = p_t^x (y_t^x - q_t^x) \tag{32}$$

The importable sector is symmetric in structure, combining capital K_t^m, high-skilled labor $H_t^{H,m}$, and formal low-skilled labor $H_t^{F,m}$:

$$y_t^m = A_t^m (K_t^m)^{\alpha_m} (H_t^{H,m})^{\kappa_m} (H_t^{F,m})^{1-\alpha_m-\kappa_m} \tag{33}$$

The corresponding first-order conditions are:

$$\alpha_m \frac{p_t^m y_t^m}{K_t^m} = R_t^m \tag{34}$$

$$\kappa_m \frac{p_t^m y_t^m}{H_t^{H,m}} = (1+\tau^H)\tilde{w}_t^{H,m} \tag{35}$$

$$(1-\alpha_m-\kappa_m)\frac{p_t^m y_t^m}{H_t^{F,m}} = (1+\tau^F)\tilde{w}_t^{F,m} \tag{36}$$

The shortfall of domestic importable production relative to domestic demand is covered by imports, whose value is:

$$m_t = p_t^m(q_t^m - y_t^m) \tag{37}$$

5.3.4 Non-tradable goods

The non-tradable sector differs from the tradable sectors in two respects. First, it is the only sector that employs informal low-skilled labor. Second, the low-skilled labor input $H_t^{L,n}$ is not a simple factor but a CES composite of formal low-skilled workers $H_t^{F,n}$ and informal low-skilled workers $H_t^{I,n}$, with elasticity of substitution θ_L and expenditure share ω_L on formal labor. The production function is therefore:

$$y_t^n = A_t^n (K_t^n)^{\alpha_n} (H_t^{H,n})^{\kappa_n} (H_t^{L,n})^{1-\alpha_n-\kappa_n} \tag{38}$$

where the composite labor input is defined by:

$$H_t^{L,n} = \left[\omega_L^{\frac{1}{\theta_L}}\left(H_t^{F,n}\right)^{\frac{\theta_L-1}{\theta_L}} + (1-\omega_L)^{\frac{1}{\theta_L}}\left(H_t^{I,n}\right)^{\frac{\theta_L-1}{\theta_L}}\right]^{\frac{\theta_L}{\theta_L-1}} \tag{39}$$

Profit maximization yields first-order conditions for capital, high-skilled labor, formal low-skilled labor, and informal low-skilled labor. Capital and high-skilled labor conditions are analogous to those in the tradable sectors. For the two types of low-skilled labor, the optimality conditions follow from differentiating through the CES composite and reflect the key asymmetry of the model: formal low-skilled labor faces the payroll tax wedge $(1+\tau^F)$, whereas informal labor is exempt:

$$\alpha_n \frac{p_t^n y_t^n}{K_t^n} = R_t^n \tag{40}$$

$$\kappa_n \frac{p_t^n y_t^n}{H_t^{H,n}} = (1+\tau^H)\tilde{w}_t^{H,n} \tag{41}$$

$$(1-\alpha_n-\kappa_n)\frac{p_t^n y_t^n}{H_t^{L,n}}\,\omega_L^{\frac{1}{\theta_L}}\left(\frac{H_t^{L,n}}{H_t^{F,n}}\right)^{\frac{1}{\theta_L}} = (1+\tau^F)\tilde{w}_t^{F,n} \tag{42}$$

$$(1-\alpha_n-\kappa_n)\frac{p_t^n y_t^n}{H_t^{L,n}}\,(1-\omega_L)^{\frac{1}{\theta_L}}\left(\frac{H_t^{L,n}}{H_t^{I,n}}\right)^{\frac{1}{\theta_L}} = \tilde{w}_t^{I,n} \tag{43}$$

Finally, non-tradable goods are not internationally traded, so domestic production must equal domestic demand in every period:

$$y_t^n = q_t^n \tag{44}$$

5.4 Macroeconomic Equilibrium and Shocks

Goods market clearing. Non-tradable goods market clearing requires that domestic production equals domestic demand: $y_t^n = q_t^n$. Exportable goods produced in excess of domestic demand are exported, and the shortfall of importable domestic production relative to domestic demand is met by imports. These net trade flows determine the current account together with interest payments on the external debt.

Fiscal rule. The government collects payroll taxes on all formal employment across sectors and household types, and rebates the proceeds as lump-sum transfers to low-skilled households:

$$T_t = \tau^H \sum_{j\in\{x,m,n\}} H_t^{H,j} + \tau^F \sum_{j\in\{x,m,n\}} H_t^{F,j}. \tag{45}$$

This balanced-budget rule keeps the fiscal block revenue-neutral in every period, so the model focuses on the allocative distortion introduced by the payroll tax wedge rather than on fiscal multiplier effects. Because transfers accrue only to low-skilled households, a terms-of-trade expansion that raises formal employment also raises T_t, partially offsetting the compression in low-skilled real wages during the reallocation.

External balance. Net foreign assets are held exclusively by high-skilled households. The balance of payments identity equates the change in the economy's net foreign asset position to the trade balance:

$$p_t^{\tau} B_t^* - p_t^{\tau}(1 + r_{t-1}^*) B_{t-1}^* = x_t - m_t. \tag{46}$$

A trade surplus ($x_t > m_t$) allows the economy to accumulate foreign assets; a deficit requires external borrowing at the endogenous interest rate r_t^*. The country risk premium in equation (10) ensures that the net foreign asset position is stationary around its calibrated steady-state value B_{ss}^*.

Terms-of-trade shock. The terms of trade is defined as the ratio of the export price to the import price,

$$tot_t = \frac{p_t^x}{p_t^m}, \tag{47}$$

and follows a stationary AR(1) process around its steady-state value of unity:

$$tot_t = (tot_{t-1})^{\rho_{tot}} (tot_{ss})^{1-\rho_{tot}} (1 + \varepsilon_t^{tot}). \tag{48}$$

The persistence parameter $\rho_{tot} = 0.90$ is estimated from Colombian data on the Brent oil price and captures the empirically observed slow mean-reversion of commodity price shocks. The innovation ε_t^{tot} is i.i.d. with standard deviation $\sigma_{tot} = 0.075$, calibrated to match the unconditional volatility of the Colombian terms of trade. High persistence is quantitatively important: it implies that a positive terms-of-trade realization shifts agents' entire wealth path upward, generating a consumption response that exceeds the contemporaneous income gain and sustaining the informality expansion well beyond the period of impact.

Sectoral productivity. Each sector's total factor productivity follows an independent AR(1) process:

$$A_t^j = (A_{t-1}^j)^{\rho_{aj}} (A_{ss}^j)^{1-\rho_{aj}} (1 + \varepsilon_t^{aj}), \qquad j \in \{x, m, n\}. \tag{49}$$

In the baseline experiments the TFP innovations are set to zero, so the model is driven exclusively by the external terms-of-trade shock. The sectoral TFP levels A_{ss}^j are calibration targets, set to match the steady-state output shares and trade balance ratio.

6 Calibration

The calibration proceeds in two stages. In the first stage, a subset of parameters is assigned directly from the literature or from Colombian data moments that can be identified independently of the model's steady-state equilibrium conditions. In the second stage, eight parameters are determined jointly by requiring that the non-stochastic steady state satisfies twelve equilibrium conditions and seven empirical targets drawn from Colombian national accounts and labor market surveys. The model is normalized so that steady-state GDP equals one. The baseline preference and technology parameters follow Schmitt-Grohé and Uribe (2018), who estimate a three-sector small open economy model for a panel of 38 emerging economies; departures from that benchmark are motivated by features of the Colombian economy and by the labor market heterogeneity introduced in our framework. Additional support for the Colombian-specific parameterization is provided by González et al. (2014), who develop a DSGE fiscal model for Colombia with non-Ricardian households, wage rigidities, and oil price shocks. Table 2 reports all parameter values.

Externally Calibrated Parameters

Preferences and households. The household discount factor $\beta = 0.99$ implies a steady-state world interest rate of $r^* = 1/\beta - 1 = 0.0101$ per quarter (approximately 4.1 percent annually), consistent with González et al. (2014) and standard in the emerging-economy business-cycle literature. The inverse of the intertemporal elasticity of substitution is $\sigma = 2$, following Schmitt-Grohé and Uribe (2018) and González et al. (2014), who follow López-Piñeros (2001) for Colombia. The share of H-type (Ricardian) households in the population is $N = 0.52$, calibrated to Colombian data (see **?**). Labor disutility weights are normalized to $\psi_H = \psi_F = 1$; the corresponding parameter for informal workers, $\psi_I = 0.23$, is determined jointly in the second stage. The inverse Frisch elasticity is $\eta_H = 1.0$ for high-skilled workers and $\eta_L = 0.5$ for low-skilled workers (Frisch elasticities of 1 and 2, respectively). The value of $\eta_L = 0.5$ is consistent with the estimate of 0.5 used in González et al. (2014) following Prada-Sarmiento and Rojas (2009) for Colombia, and close to the implied Frisch elasticity of 2.2 in Schmitt-Grohé and Uribe (2018) following Mendoza (1991).

Capital and foreign assets. Capital depreciates at the quarterly rate $\delta = 0.025$ (10 percent annually), following Schmitt-Grohé and Uribe (2018). Capital accumulation is subject to convex adjustment costs with parameter $\Phi_j = 15$ in all three sectors. External bond holdings are stabilized by a debt-elastic interest rate with parameter $\Phi_B = 0.001$, consistent with González et al. (2014) and small enough not to distort high-frequency dynamics (Gertler et al., 2007). The steady-state net foreign asset position, $B^*_{ss} = 4.95$, is determined by the current account target (see below).

Fiscal. Payroll tax rates are set symmetrically at $\tau^H = \tau^F = 0.10$, consistent with statutory employer contribution rates for social security in Colombia (see **?**). Informal workers are exempt from payroll taxes, so the gross formal–informal labor cost differential is $(1 + \tau^F) = 1.10$.

Wage rigidities. All labor markets are subject to Rotemberg wage rigidities. The elasticity of substitution across labor varieties is $\theta_j = 5$ in all seven markets, implying a steady-state wage markup of 25 percent. The Rotemberg adjustment cost is $\phi_j = 10$ for all formal labor markets and $\phi_I = 5$ for the informal market, so informal wages adjust roughly twice as fast as formal wages following a shock.

Final good aggregator. The CES expenditure share of tradables in the final good, $\omega_c = 0.381$, is determined jointly in the second stage (see below). The tradable/non-tradable elasticity of substitution is $\theta_c = 0.5$, following Schmitt-Grohé and Uribe (2018), who follow the survey by Akinci (2011). The share of exportables in the tradable composite is $\omega_\tau = 0.40$, and the exportable/importable elasticity of substitution is $\theta_\tau = 0.5$, below the unit elasticity in Schmitt-Grohé and Uribe (2018) but consistent with quarterly-frequency estimates for emerging economies (Corsetti et al., 2008).

Technology. Sectoral capital shares follow Schmitt-Grohé and Uribe (2018) for the exportable sector ($\alpha_x = 0.35$) and are adjusted for Colombian data in the importable and non-tradable sectors ($\alpha_m = 0.30$, $\alpha_n = 0.20$), reflecting the higher labor intensity of domestic-oriented production. The H-skilled labor share in the two tradable sectors is $\kappa_x = \kappa_m = 0.50$. The CES weights governing labor composition in the non-tradable sector ($\omega_n = 0.476$, $\omega_L = 0.393$) and the sectoral TFP levels ($A^x_{ss} = 0.577$, $A^m_{ss} = 0.358$, $A^n_{ss} = 0.491$) are all determined jointly in the second stage. The H/low-skilled substitution elasticity is $\theta_n = 1.01$ (approximately Cobb-Douglas) and the formal/informal elasticity is $\theta_L = 1.50$.

Shock. The AR(1) persistence of the terms-of-trade shock is $\rho_{tot} = 0.90$, estimated from quarterly Colombian data (, XX: period, source). Steady-state prices are normalized so that $\text{tot}_{ss} = 1$ and $p_c = 1$.

Jointly Calibrated Parameters

The second stage solves for eight unknowns by requiring that the steady state satisfies twelve equilibrium conditions and seven empirical targets simultaneously. The unknowns are: the CES tradable share ω_c; the H-skilled share in the non-tradable labor composite ω_n; the formal share in the low-skilled composite ω_L; the informal labor disutility ψ_I; the sectoral TFP levels A^x_{ss}, A^m_{ss}, A^n_{ss}; and the steady-state net foreign asset position B^*_{ss}. The remaining eleven unknowns (per-capita capital stocks k_{Hx}, k_{Hm}, k_{Hn}; consumption levels c_H, c_L; and sector-level wages w_{Hx}, w_{Hm}, w_{Hn}, w_{Fx}, w_{Fm}, w_{Fn}, w_{In}) are determined residually by the equilibrium conditions once the eight parameters above are pinned down by the targets.

The equilibrium conditions impose profit maximization and market clearing. For each sector $j \in \{x, m, n\}$, the capital rental condition requires

$$\frac{\alpha_j \, p_j \, y_j}{K_j} = R_k \equiv \left(\tfrac{1}{\beta} - (1-\delta)\right) p_c, \tag{50}$$

which pins down sectoral capital stocks given output and prices. Firm first-order conditions for formal labor equate each type's value of marginal product to the tax-inclusive wage:

$$\frac{\kappa_j \, p_j \, y_j}{H_{Hj}} = (1+\tau^H)\, w_{Hj}, \qquad \frac{(1-\alpha_j-\kappa_j)\, p_j \, y_j}{H_{Fj}} = (1+\tau^F)\, w_{Fj}, \qquad j \in \{x, m\}, \tag{51}$$

with analogous CES-weighted conditions for the non-tradable sector. For informal labor there is no payroll tax:

$$\frac{(1-\alpha_n)\, p_n \, y_n}{L_n} \left(\frac{(1-\omega_n)\, L_n}{H_{Ln}}\right)^{1/\theta_n} \left(\frac{(1-\omega_L)\, H_{Ln}}{H_{In}}\right)^{1/\theta_L} = w_{In}, \tag{52}$$

whereas the formal low-skilled condition has $(1+\tau^F)\, w_{Fn}$ on the right-hand side, making the payroll tax wedge the primary structural source of the formal–informal wage gap at the calibrated steady state. The remaining conditions impose non-tradable market clearing ($y_n = q_n$) and the hand-to-mouth budget constraint for the L household ($p_c \, c_L = \sum_s w_{Fs} \, h_{Fs} + w_{In} \, h_{In} + T/(1-N)$, where T is the revenue-neutral fiscal transfer).

Calibration Targets

Table 1 reports the seven empirical targets. Four characterize Colombia's macroeconomic structure: a GDP normalization, a trade balance of -5 percent of GDP, trade openness of 35 percent, and a non-tradable output share of 65 percent. These depart from the cross-country averages in Schmitt-Grohé and Uribe (2018) (non-tradable share of 50 percent, trade balance of +1 percent) and are calibrated to Colombian data over [XX: period].

Three additional targets discipline the labor market structure, for which there is no counterpart in Schmitt-Grohé and Uribe (2018) because that model has a single representative household with undifferentiated labor. To construct the labor-market targets, we classify workers by educational attainment. High-skill workers are those who have completed tertiary or university education, while low-skill workers are those whose highest educational attainment is upper secondary education or below. Within each skill group, we further distinguish between formal and informal workers, yielding formal high-skill workers and formal and informal low-skill workers. We calibrate the wage ratios and the informal employment share to values close to their corresponding averages in the GEIH over 2008–2019. Accordingly, the ratio of average high-skilled to average formal low-skilled wages is set to 1.5, pinning down ω_n. The ratio of average formal to informal low-skilled wages is set to 1.4, which, together with the tax wedge $(1 + \tau^F) = 1.10$, identifies ω_L. Finally, the share of informal employment in total hours is set to 40 percent and pins down $\psi_I = 0.23$.

Table 1: Calibration targets

Target	Expression	Value	Source
GDP normalization	$\text{GDP} = p_c(C + I) + \text{NX}$	1	Normalization
Trade Balance	NX/GDP	−5%	DANE
Trade openness	$(X + M)/\text{GDP}$	35%	DANE
Non-tradable share	$p_n y_n/\text{GDP}$	65%	DANE
H/F wage ratio	$\bar{w}_H/\bar{w}_F$	1.5	GEIH
F/I wage ratio	$\bar{w}_F/w_{In}$	1.4	GEIH
Informal emp. share	H_{In}/H_{total}	40%	GEIH

Notes: $\bar{w}_H$ and $\bar{w}_F$ are employment-weighted average wages across sectors. Labor market targets are estimated from the Gran Encuesta Integrada de Hogares (GEIH, DANE). Macroeconomic aggregates are from Colombian national accounts (DANE) and the external sector accounts (Banco de la República).

The system is solved numerically using `fsolve` in MATLAB, and all equilibrium conditions hold to numerical precision at the reported steady state.

Table 2: Model parameters.

Parameter	**Description**	**Value**	**Source**
Preferences and households			

(continued on next page)

(continued from previous page)

Parameter	Description	Value	Source
β	Discount factor	0.99	González et al. 2014
σ	Inverse of the intertemporal EoS	2.0	González et al. 2014
N	Share of high-skilled households	0.52	Data
ψ_H	Disutility weight of labor (high-skilled)	1.0	Normalization
η_H	Inverse Frisch elasticity (high-skilled)	1.0	[XX]
ψ_F	Disutility weight of formal labor (low-skilled)	1.0	Normalization
ψ_I	Disutility weight of informal labor (low-skilled)	0.23	Calibration
η_L	Inverse Frisch elasticity (low-skilled)	0.5	González et al. 2014
Capital and external assets			
δ	Capital depreciation rate	0.025	Schmitt-Grohé and Uribe 2018
Φ_j	Capital adjustment cost, sector $j \in \{m, x, n\}$	15	Calibrated
Φ_B	Country risk premium	0.001	González et al. 2014
r^*_{ss}	Steady-state world interest rate	0.0101	González et al. 2014
B^*_{ss}	Steady-state net foreign asset position	4.95	Calibrated
Fiscal			
τ^H	Payroll tax on high-skilled labor	0.1	Data
τ^F	Payroll tax on formal low-skilled labor	0.1	Data
Wage rigidities (Rotemberg)			
θ_j	EoS across labor varieties of type j	5.0	Literature

(continued on next page)

(continued from previous page)

Parameter	Description	Value	Source
$\phi_{j\neq I}$	Wage adjustment cost of type $j \neq I$	10	Calibrated
ϕ_I	Wage adjustment cost of type I	5	Calibrated
Final good aggregator (CES)			
ω_c	Expenditure share of tradables in the final good	0.3810	Calibrated
θ_c	Tradable / non-tradable EoS	0.5	Schmitt-Grohé and Uribe 2018
ω_τ	Share of exportables in the tradable good	0.4	[XX]
θ_τ	Exportable / importable EoS	0.5	Corsetti et al. 2008
Technology — exportable and importable sectors			
α_x	Capital share, exportable sector	0.35	Schmitt-Grohé and Uribe 2018
κ_x	High-skilled labor share, exportable	0.50	Data
α_m	Capital share, importable sector	0.30	Data
κ_m	High-skilled labor share, importable	0.50	Data
Technology — non-tradable sector			
α_n	Capital share, non-tradable sector	0.20	Data
ω_n	Share of high-skilled labor in L^n	0.4756	Calibrated
θ_n	High- / low-skilled EoS	1.01	[XX]
ω_L	Share of formal labor in $H^{L,n}$	0.3933	Calibrated
θ_L	Formal / informal EoS	1.50	[XX]
Shocks			
ρ_{tot}	Persistence of the terms-of-trade shock	0.9	Data

(continued on next page)

(continued from previous page)

Parameter	Description	Value	Source
tot_{ss}	Steady-state terms of trade	1.0	Normalization
A_{ss}^{x}	Steady-state TFP, sector x	0.5769	Calibrated
A_{ss}^{m}	Steady-state TFP, sector m	0.3580	Calibrated
A_{ss}^{n}	Steady-state TFP, sector n	0.4906	Calibrated

7 Results

Figure 5 reports the impulse response functions of the main macroeconomic aggregates to a one-standard-deviation positive shock to the terms of trade. The shock follows an AR(1) process, peaking at approximately 8 percent above steady state on impact and dissipating smoothly over subsequent periods. As documented by Schmitt-Grohé and Uribe 2018, a positive terms-of-trade shock raises the relative price of exportables, acting as a positive income shock that expands aggregate economic activity — output, consumption, and the trade balance. At the same time, it reduces the relative price of importable goods, triggering a sectoral reallocation: resources shift toward exportable and non-tradable goods and away from importables. On the tradable margin, the mechanism is direct — higher exportable prices raise profitability in that sector while reducing the incentive to produce importable goods domestically, so importable output contracts and is replaced by imports from the rest of the world. On the non-tradable margin, the mechanism is a pure income effect: as households have more resources, they increase consumption of both tradable and non-tradable goods, raising demand for labor and capital in the non-tradable sector while reducing it in the importable sector.

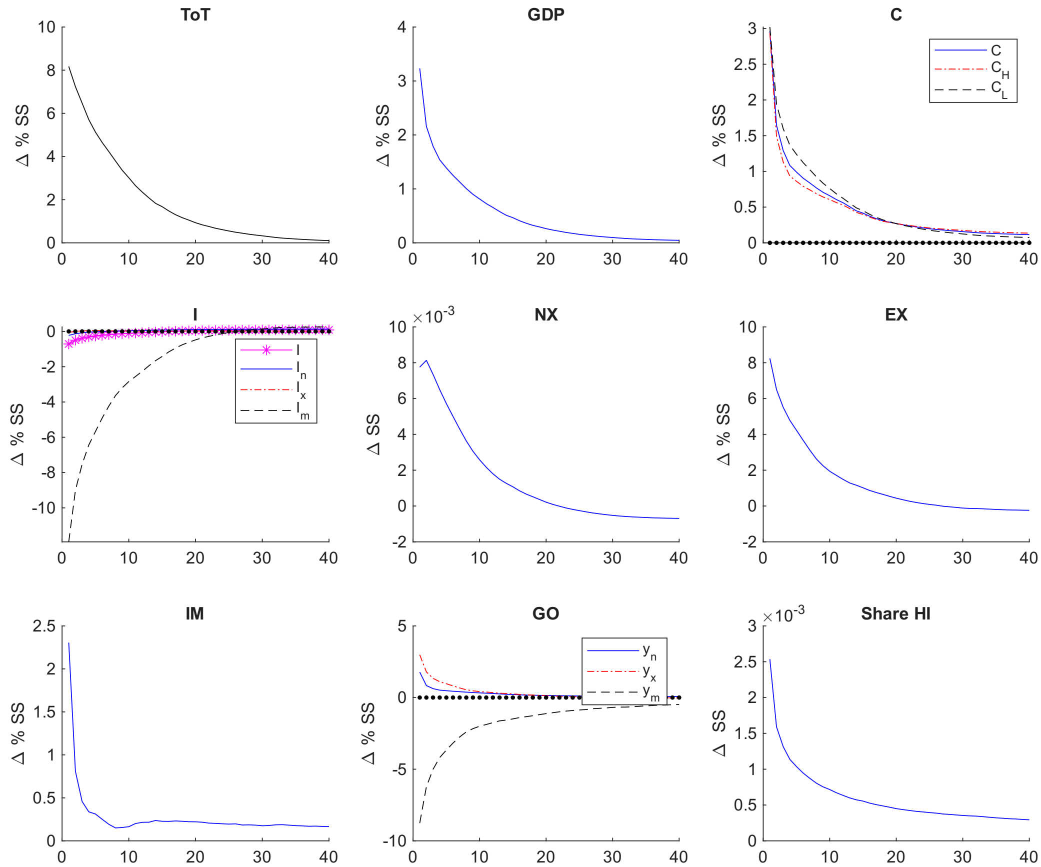

Figure 5: Impulse response functions to a ToT shock.

These dynamics are quantitatively consistent with the mechanisms above. GDP rises approximately 3.2 percent on impact and decays monotonically, tracking the terms-of-trade path closely. Consumption increases for both household types — C_H and C_L — peaking around 2.8–3 percent above steady state. Notably, the two responses are nearly identical on impact despite only high-skilled households holding assets: low-skilled households benefit through higher formal and informal wages, which translate directly into consumption. During the transition, higher investment needs cause high-skilled consumption to decline somewhat faster.

Investment reveals a pronounced asymmetry that directly reflects the reallocation mechanism. The dominant movement is a sharp contraction in importable-sector investment (I_m), which falls approximately 10–11 percent on impact before slowly recovering, while investment in the exportable and non-tradable sectors barely moves — consistent with capital fleeing the importable sector as its relative price deteriorates. On the external side, exports jump roughly 8 percent on impact and decay gradually, while imports rise initially (about 2.3 percent) before falling back. The trade balance turns positive on impact and slightly negative in the medium run, consistent with a standard current account response: the favorable shock generates a short-run surplus that is gradually unwound as domestic absorption rises and the shock dissipates. The initial increase in imports reflects both higher final-goods demand — since final production requires imported inputs — and the income-driven rise in household consumption of tradables.

The sectoral reallocation is clearest in gross output. Exportable output (y_x) and non-tradable production (y_n) increase, while importable output (y_m) collapses by roughly 9–10 percent on impact before recovering. Even amid this contraction, the informality share rises slightly on impact (approximately 2.5×10^{-3} in levels) and then decays. This positive response reflects the mild income-driven expansion of the non-tradable sector — where informal labor operates — which is large enough to offset the employment losses in the importable sector. A key amplification mechanism for this labor reallocation is GHH preferences. Because GHH preferences eliminate the wealth effect on labor supply, the wage increases in the non-tradable sector — both formal and informal — translate directly into a substitution toward that sector, reinforcing the expansion of informal employment in the aggregate.

Regarding labor market outcomes, Figure 6 reports the corresponding impulse responses for all sectors, consistent with the reallocation described above. In the exportable sector, the strong demand pull from the higher relative price draws in labor sharply: high-skilled (H_{Hx}) and formal low-skilled (H_{Fx}) employment increase by approximately 5.5 and 2.5 percent on impact, respectively. Wages in this sector exhibit a pronounced hump shape, peaking around periods 3–5 — a direct implication of Rotemberg adjustment costs slowing the pass-through of higher exportable prices to wages.

In the non-tradable sector, the income effect drives a more moderate expansion. All three labor types increase on impact, with high-skilled labor (H_{Hn}) rising the most (approximately 2.8 percent), followed by formal low-skilled (H_{Fn}, around 2.3 percent) and informal labor (H_{In}, around 1.8 percent). Wages across all types also rise; high-skilled wages peak highest (around 0.75 percent), while formal low-skilled wages display a mild hump-shaped response, again reflecting Rotemberg rigidities.

The most striking responses occur in the importable sector, where the collapse in relative price translates into large employment and wage losses. Formal low-skilled employment (H_{Fm}) falls approximately 13–14 percent and high-skilled employment (H_{Hm}) around 11–12 percent on impact. Wages also decline — w_{Hm} by around 1.5 percent and w_{Fm} by around 0.7 percent — and follow a U-shaped recovery before converging to steady state.

Taken together, the impulse responses tell a coherent story: the terms-of-trade improvement raises the relative price of exportables, pulling labor and capital toward that sector, collapsing importable-sector activity, and generating an income effect that mildly expands the non-tradable sector and its informal labor margin.

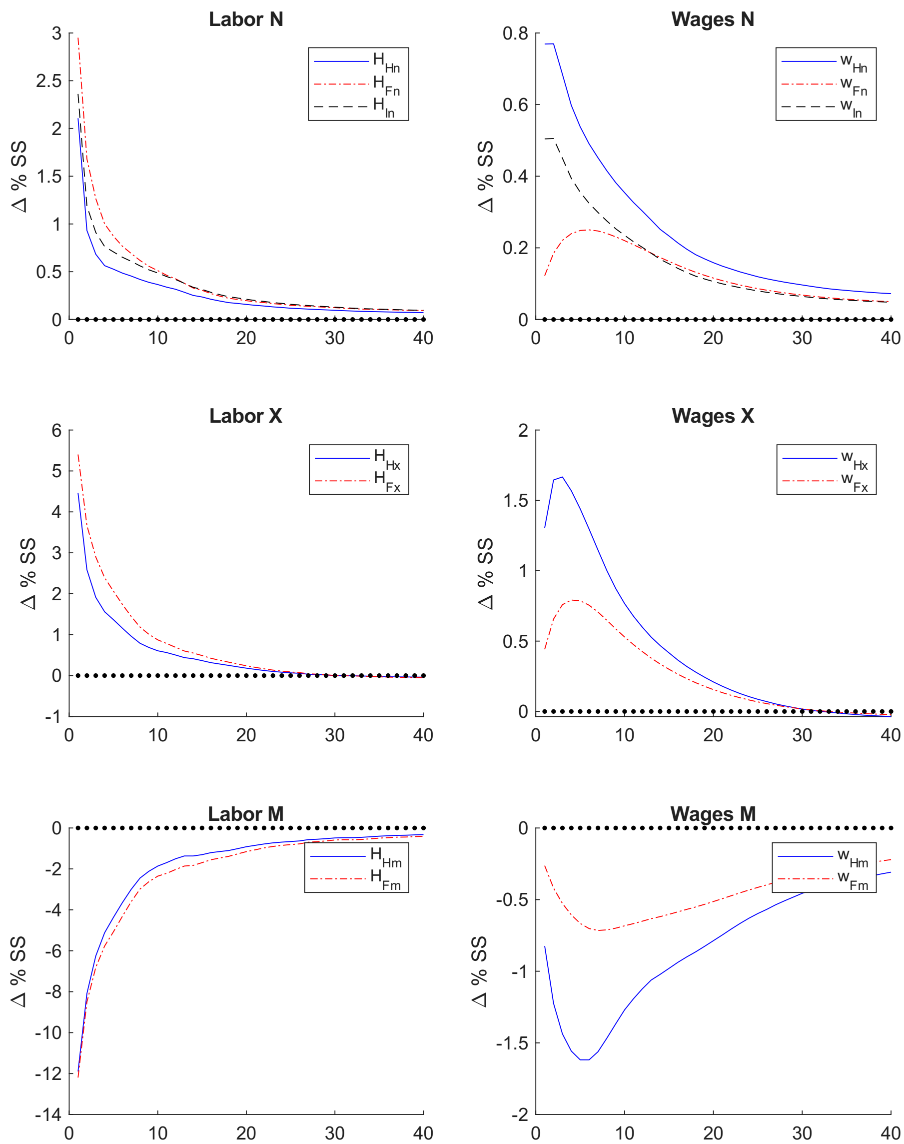

Figure 6: Labor market responses to a ToT shock.

7.1 Sensitivity Analysis

This section examines the robustness of the benchmark results along two dimensions: alternative modeling assumptions and labor market rigidities. In both cases, the model is recalibrated to match the same aggregate targets as the benchmark.

7.2 Model Assumptions

The first exercise considers two variants of the benchmark. The first replaces GHH preferences with separable utility, which reintroduces the standard wealth effect on labor supply. The second retains GHH preferences but collapses the two household types into a single representative agent that simultaneously supplies all labor types — high-skilled, formal low-skilled, and informal. Each variant is recalibrated separately to match the same aggregate targets as the benchmark.

Figures 7 and 8 compare the impulse responses of the model across three specifications: the benchmark (GHH preferences, two heterogeneous households, solid blue), separable preferences with two households (dashed green), and GHH preferences with a single representative household (dotted red). The comparison isolates the contribution of each modeling choice to the informality result.

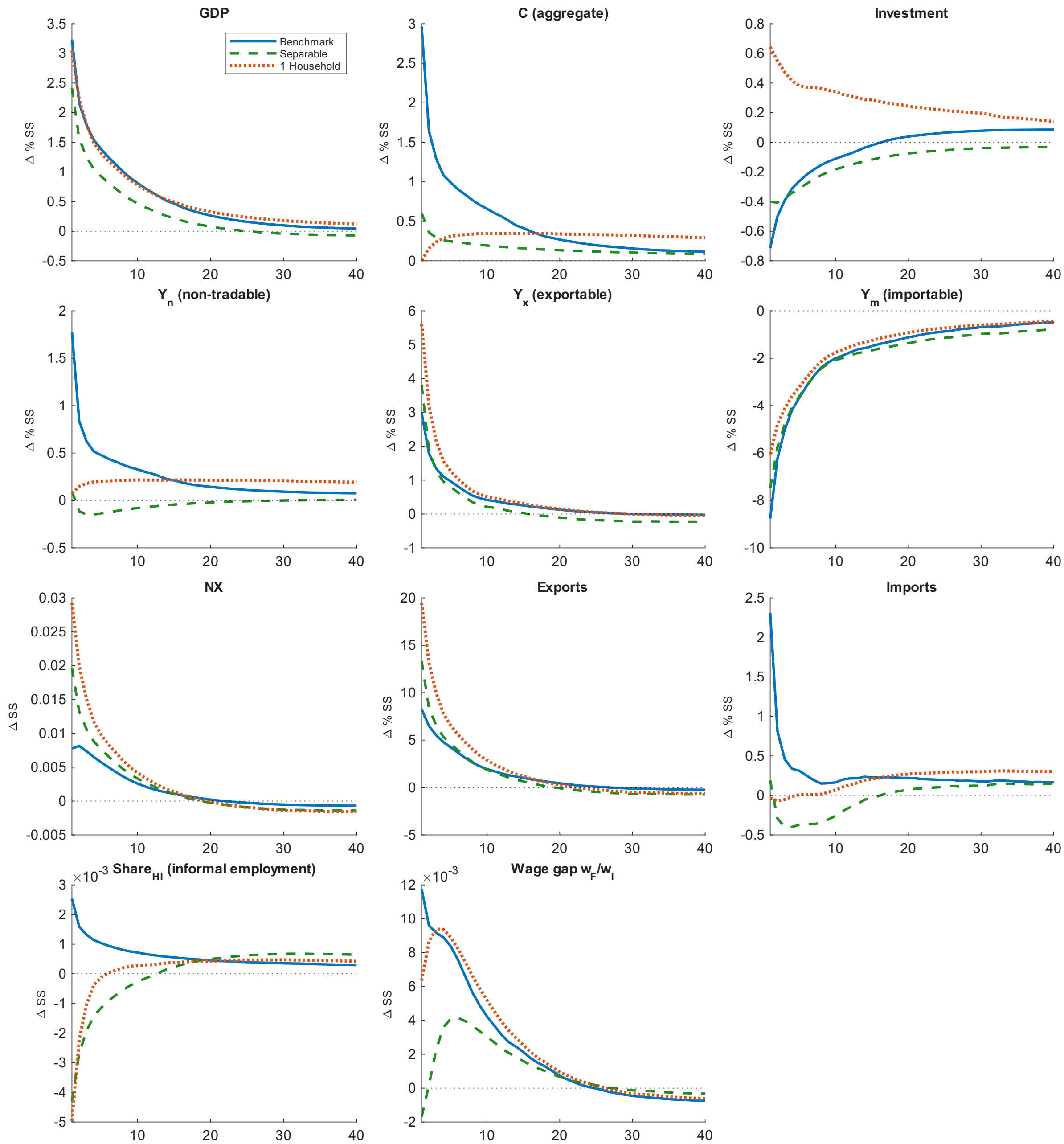

Figure 7: Sensitivity analysis: macroeconomic aggregates. Solid blue: benchmark (GHH, 2HH). Dashed green: separable preferences (2HH). Dotted red: representative agent (GHH, 1HH).

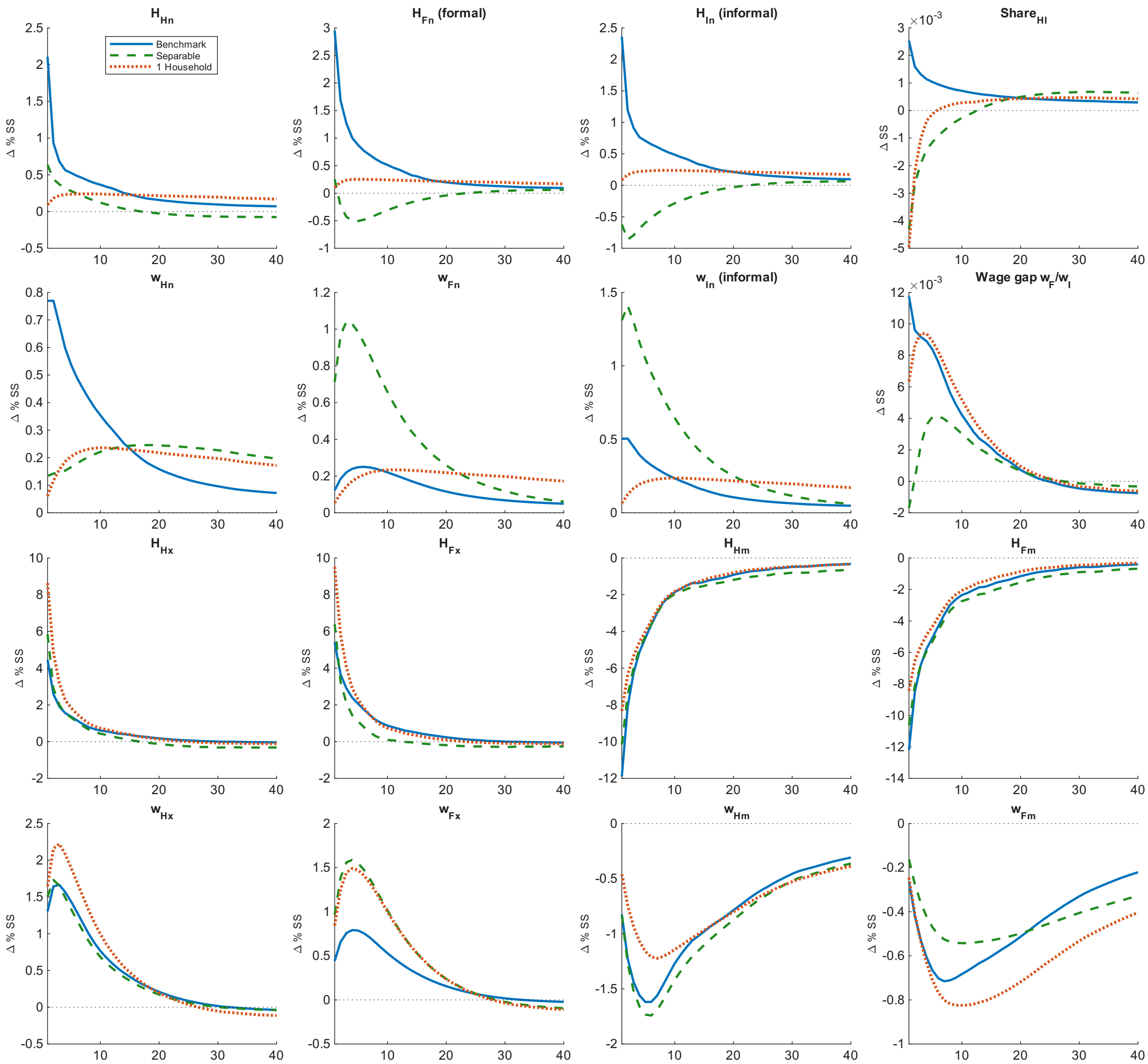

Figure 8: Sensitivity analysis: labor market. Solid blue: benchmark (GHH, 2HH). Dashed green: separable preferences (2HH). Dotted red: representative agent (GHH, 1HH).

Aggregate dynamics. The three specifications agree qualitatively on the tradable sector: exportable output rises by roughly 5–6 % under the benchmark and 3 percent under the two alternative specifications, importable output contracts by approximately 8–9 % in all cases, and the trade and current account responses are broadly similar. Both exports and imports increase, and the trade balance improve. The main aggregate differences operate through consumption and investment. Under the benchmark, aggregate consumption rises by approximately 3.0 % on impact, reflecting the amplified income effect generated by GHH preferences; under separable preferences and the representative-agent specification, consumption rises by only about 0–0.6 %. Investment also differs qualitatively: both the benchmark and the separable model generate a decline in aggregate investment on impact (around 0.7 %), driven primarily by the contraction in importable-sector investment, and the bigger expansion of consumption. By contrast, the representative household responds to the terms-of-trade shock by increasing total investment (roughly 0.4 % on impact), as the improved return to capital in the exportable sector induces a reallocation of savings toward capital accumulation rather than current consumption. In other words, the hand-to-mouth agent offsets the expansionary effect on investment by reducing the share of agents that have incentives to save and smooth consumption.

The non-tradable channel and informality. The non-tradable sector is the locus of the key disagreement across specifications. Under the benchmark, non-tradable output expands by approximately 1.8 % on impact, and all three types of non-tradable employment—H_{Hn}, H_{Fn}, and H_{In}—rise by roughly 2–2.5 %. The share of informal employment (Share_{HI}) increases by approximately 2.5×10^{-3} on impact and remains positive throughout the transition.

Under separable preferences, the picture is reversed. The wealth effect on labor supply contracts non-tradable employment across all types, with H_{Fn} and H_{In} falling by about 0.9 % on impact and non-tradable output barely moving. Share_{HI} turns sharply negative (-4 to -5×10^{-3} on impact), as the reduction in informal labor supply dominates the modest change in demand.

With the representative-agent specification, the informality response also turns negative (approximately -4×10^{-3} on impact), despite GHH preferences that eliminate the intratemporal wealth effect on labor supply. Non-tradable output rises by only about 0.1 %—a fraction of the benchmark response—and informal employment rises negligibly. The driving mechanism is the investment channel: the representative household, owning all capital, reallocates saving toward the exportable sector in response to higher capital returns, which diverts resources away from non-tradable production and attenuates the non-tradable expansion. In addition, the single household internalizes the increase in wealth from capital appreciation and does not face the binding income constraint that forces low-skilled hand-to-mouth households to respond to wage incentives in the non-tradable sector.

Two failures for different reasons. The separable and representative-agent specifications both generate counterfactual declines in informality, but through distinct channels. Under separable preferences, the failure is intratemporal: the standard income effect induces households to reduce labor supply contemporaneously, preventing non-tradable production from expanding in response to higher consumption demand. Under the representative-agent specification, the failure is distributional: even with GHH preferences, the single household simultaneously owns capital and supplies informal labor, so the ToT-induced increase in capital income weakens the incentive to supply informal labor in the non-tradable sector. Household heterogeneity is therefore not merely a refinement of the model but a structural requirement for replicating the empirical informality dynamics: the hand-to-mouth constraint of low-skilled households is the mechanism through which higher non-tradable wages translate into expanded informal employment.

Wage dynamics. Figure 8 also reveals informative differences in wages. In the non-tradable sector, the benchmark generates the largest high-skilled wage response (w_{Hn} rises about 0.8 %), consistent with the strong demand expansion. Under separable preferences, formal and informal wages in non-tradable (w_{Fn} and w_{In}) show a pronounced hump-shaped response, peaking around 1–1.4 % before returning to steady state. This seemingly paradoxical combination of rising wages and falling employment reflects a supply-side contraction: firms demand roughly the same labor but households are less willing to supply it, so wages must rise to clear the market even as equilibrium employment falls. The wage gap w_F/w_I widens on impact in all specifications (most sharply under the benchmark, 10×10^{-3}), as the ToT shock raises formal sector wages faster than informal wages, before narrowing and eventually turning slightly negative at longer horizons.

7.2.1 No Wage Rigidities

The second exercise considers no-wage rigidities. Figure 9 reports the impulse responses of aggregate variables, household consumption, and informality to a one-standard-deviation positive terms-of-trade shock under the rigid-wage (solid blue) and flexible-wage (dashed red) regimes. Figure 10 shows the corresponding responses in the labor market, organized by sector.

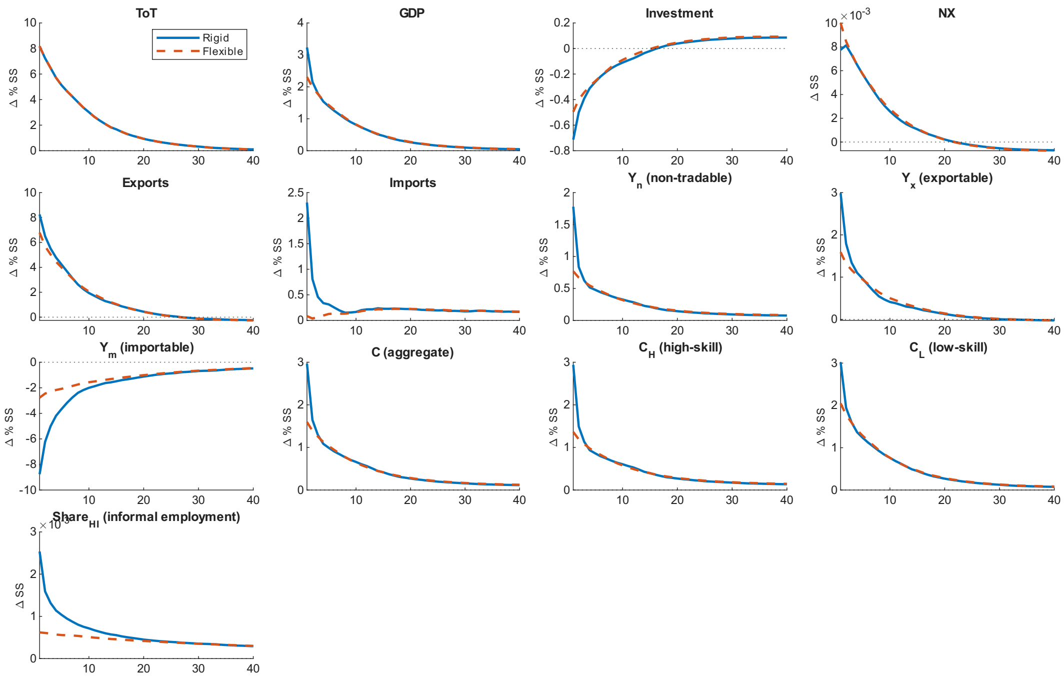

Figure 9: Impulse response functions: rigid vs. flexible wages — macroeconomic aggregates.

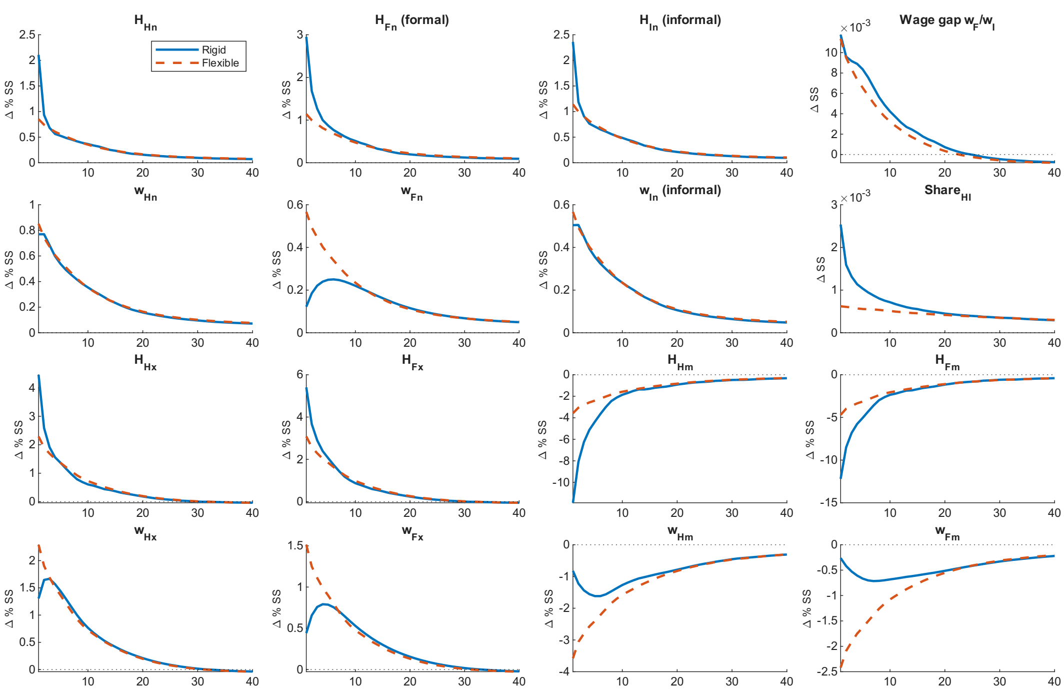

Figure 10: Impulse response functions: rigid vs. flexible wages — labor market.

Aggregate dynamics. The positive terms-of-trade shock raises the relative price of exportable goods, generating a wealth transfer to the domestic economy. On impact, GDP expands by approximately 3–4 % relative to its steady-state value and returns to baseline gradually, consistent with the high persistence of the shock ($\rho_{tot} = 0.90$). Export volumes rise sharply on impact, closely tracking the terms-of-trade improvement, while import volumes fall slightly, producing a marked improvement in net exports. Investment declines on impact, reflecting the short-run reallocation of resources away from capital accumulation toward consumption and tradable production. Aggregate consumption and household-level consumption for both high-skill (c^H) and low-skill (c^L) households rise by roughly 3 %, and the responses are nearly identical across wage regimes. This finding indicates that, at the aggregate level, wage rigidity does not substantially alter the macroeconomic transmission of a terms-of-trade shock.

Sectoral output. The reallocation of resources across sectors is asymmetric. Output in the exportable sector (y^x) expands, benefiting directly from the higher relative price of exports. Non-tradable output (y^n) also rises moderately, driven by the income effect on domestic demand. By contrast, importable output (y^m) contracts, as the terms-of-trade improvement lowers the domestic relative price of importable goods, reducing the profitability of import-competing production. This contraction is more pronounced under rigid wages: y^m falls by approximately 10 % on impact in the rigid-wage economy, compared with roughly 7–8 % under flexible wages. This difference constitutes the primary macroeconomic distinction between the two regimes.

Labor market reallocation. Figure 10 reveals that the aggregate employment responses mask important sectoral heterogeneity. Employment expands in both the non-tradable and exportable sectors, as firms in those sectors absorb workers released from the contracting importable sector. The reallocation is strongest for formal low-skill workers ($H_t^{F,x}$), whose employment in the exportable sector rises by up to 6 % on impact.

In the importable sector, the contrast between regimes is striking. Under rigid wages, formal employment falls by approximately 10 % ($H_t^{H,m}$) and 15 % ($H_t^{F,m}$) on impact, whereas under flexible wages the corresponding declines are considerably smaller. This pattern reflects the central mechanism of the model: when wages cannot adjust freely, firms in the contracting importable sector cannot reduce labor costs and instead adjust along the employment margin, amplifying job losses relative to the flexible-wage benchmark.

Wages. Wage dynamics also differ across sectors. In the exportable and non-tradable sectors, nominal wages rise, consistent with increased labor demand. Under flexible wages, wages in these sectors exhibit a hump-shaped response—rising rapidly on impact and then declining as the shock dissipates—while under rigid wages the adjustment is more gradual due to Rotemberg costs. In the importable sector, wages ($\tilde{w}^{H,m}$ and $\tilde{w}^{F,m}$) fall in both regimes, but the decline is more persistent under rigid wages, as adjustment costs slow the return to the steady-state wage.

Informal employment. The share of informal employment ($H_t^{I,n}$) increases modestly on impact and decays smoothly thereafter. The positive response reflects increased labor demand in the non-tradable sector, which is the only sector accessible to informal workers. Importantly, both regimes generate a qualitatively similar informality response, suggesting that the informal labor market partially absorbs displaced formal workers from the importable sector and thereby dampens the aggregate welfare cost of wage rigidity.

8 Conclusions

This paper studies the relationship between terms-of-trade shocks and informal employment in a commodity-exporting small open economy. Using Colombia as the empirical laboratory, we combine a high-frequency identified VAR with an oil-supply news instrument (Känzig, 2021) and a calibrated DSGE model with labor market segmentation and heterogeneous households. The dual approach allows us to establish the empirical regularity, identify the mechanism through which it operates, and assess the structural conditions required to replicate it quantitatively.

The empirical evidence establishes a positive and persistent causal effect of oil price increases on informal employment. A 10 percent rise in the Brent price raises the terms of trade by roughly 5 percent and increases the share of informal employment by approximately 0.3 percentage points over a horizon of four to six quarters. Sectoral decomposition of the response reveals that the expansion in informality is concentrated in non-tradable activities — construction, transport, and hospitality — while manufacturing employment remains largely unchanged. This pattern is consistent with a spending-effect channel: higher commodity export revenues raise domestic income and demand, driving up activity in the non-tradable sector, which in Colombia employs a disproportionate share of informal workers.

The structural model translates this mechanism into a set of quantitative predictions. A positive terms-of-trade shock raises the relative price of exportable goods, generating a transfer of income to the domestic economy. Under GHH preferences, this income effect does not induce households to reduce labor supply, so higher demand for non-tradable goods is accommodated through output and employment expansion rather than price adjustment alone. Formal and informal employment in the non-tradable sector both rise on impact — informal employment by approximately 2.3 percent under the benchmark calibration — and the share of informal employment increases by roughly 2.5×10^{-3}. GDP rises by approximately 3.2 percent and aggregate consumption by 2.8 percent, while the importable sector contracts sharply, consistent with the standard Dutch-disease pattern.

Sensitivity analysis demonstrates that replicating the informality result requires three structural features to operate jointly. First, GHH preferences are necessary: replacing them with separable preferences reintroduces a standard intratemporal wealth effect that contracts labor supply, stifles non-tradable output, and reverses the informality response (Share_{HI} falls by 4–5 $\times 10^{-3}$ on impact). Second, household heterogeneity is necessary: collapsing the economy to a representative agent, even retaining GHH preferences, generates a counterfactual decline in informality. The single household internalizes capital income from the terms-of-trade windfall, which weakens its incentive to expand informal labor supply in the non-tradable sector; moreover, it reallocates savings toward capital accumulation, diverting resources away from non-tradable production. The hand-to-mouth constraint that prevents low-skilled households from smoothing wealth shocks is therefore not a modeling refinement but a structural requirement for the mechanism to operate. Third, nominal wage rigidities amplify the employment response by shifting adjustment toward labor quantities rather than wages, magnifying the expansion of informal employment for a given increase in non-tradable output.

Welfare implications are positive but modest. The terms-of-trade shock generates consumption-equivalent gains of 0.18 percent for high-skilled households and 0.26 percent for low-skilled households under the benchmark GHH calibration. The distributional profile is mildly pro-poor, reflecting the additional margins available to low-skilled workers: expansion of both formal and informal employment in the non-tradable sector compounds the income gain from higher wages. Gains are substantially smaller under separable preferences (0.06 and 0.08 percent, respectively), because the wealth effect on labor supply dampens the consumption response.

Several limitations and extensions merit mention. The model abstracts from endogenous formalization decisions: in the current framework, the formal–informal margin reflects demand-side differences across sectors and tax wedges, but workers do not choose between formal and informal contracts in response to changing incentives. Incorporating directed search or matching frictions over the formality margin would allow the model to speak to the transition dynamics of formalization policy. The analysis also maintains a revenue-neutral fiscal rule and leaves open the question of how targeted fiscal responses — subsidies to formal employment or reductions in payroll taxes — could mitigate the informality expansion associated with commodity booms. These extensions are left for future research.

References

Akinci, O. (2011). A note on the estimation of the atemporal elasticity of substitution between tradable and nontradable goods. *manuscript, Columbia University*.

Arguello, R. (2017). The plunge in oil prices: Sectoral and employment dynamics in colombia.

Arguello, R., Jiménez Giraldo, D. E., Torres, E. E., and Gasca, M. (2016). Dutch disease, informality and employment intensity in colombia.

Backus, D. K. and Crucini, M. J. (2000). Oil prices and the terms of trade. *Journal of international Economics*, 50(1):185–213.

Ballesteros, C. (2016). Mining and energy boom, dutch disease and informality in colombia: A dsge approach. *Center for Research in Economics and Finance (CIEF), Working Papers*, (16-23).

Baumeister, C. and Hamilton, J. D. (2019). Structural interpretation of vector autoregressions with incomplete identification: Revisiting the role of oil supply and demand shocks. *American Economic Review*, 109(5):1873–1910.

Bodenstein, M., Erceg, C. J., and Guerrieri, L. (2011). Oil shocks and external adjustment. *Journal of international economics*, 83(2):168–184.

Bodenstein, M., Kamber, G., and Thoenissen, C. (2018). Commodity prices and labour market dynamics in small open economies. *Journal of International Economics*, 115:170–184.

Broda, C. (2004). Terms of trade and exchange rate regimes in developing countries. *Journal of International economics*, 63(1):31–58.

Caldara, D., Cavallo, M., and Iacoviello, M. (2019). Oil price elasticities and oil price fluctuations. *Journal of Monetary Economics*, 103:1–20.

Camacho, M. and Perez-Quiros, G. (2014). Commodity prices and the business cycle in latin america: living and dying by commodities? *Emerging Markets Finance and Trade*, 50(2):110–137.

Cárdenas-Hurtado, C. A., Garavito-Acosta, A. L., and Toro-Córdoba, J. H. (2018). Asymmetric effects of terms of trade shocks on tradable and non-tradable investment rates: The colombian case. *Borradores de Economía*, (Borradores de Economía; No. 1043).

Chaudhuri, S. and Biswas, A. (2018). External terms-of-trade and labour market imperfections in developing countries: Theory and evidence. *Indian Growth and Development Review*, 11(1):22–33.

Chaudhuri, S. and Kar, S. (2023). Oil price shock and informal workers in dual labor markets. *Bulletin of Economic Research*, 75(4):869–879.

Corsetti, G., Dedola, L., and Leduc, S. (2008). International risk sharing and the transmission of productivity shocks. *The Review of Economic Studies*, 75(2):443–473.

Di Pace, F., Juvenal, L., and Petrella, I. (2025). Terms-of-trade shocks are not all alike. *American Economic Journal: Macroeconomics*, 17(2):24–64.

Drechsel, T. and Tenreyro, S. (2018). Commodity tailwinds and headwinds: Commodity terms of trade and business cycles in emerging economies. *Journal of International Economics*, 112:237–257.

Echavarría, J. J., González, A., López, E., and Rodríguez, N. (2012). Choques internacionales reales y financieros y su impacto sobre la economía colombiana. *Ensayos sobre política económica*, 30(69):13–66.

Edwards, S. (1988). Terms of trade, tariffs, and labor market adjustment in developing countries. *The World Bank Economic Review*, 2(2):165–186.

Edwards, S. and Yeyati, E. L. (2005). Flexible exchange rates as shock absorbers. *European economic review*, 49(8):2079–2105.

Fernández, A. and Meza, F. (2015). Informal employment and business cycles in emerging economies: The case of mexico. *Review of Economic Dynamics*, 18(2):381–405.

Fiess, N. M., Fugazza, M., and Maloney, W. F. (2010). Informal self-employment and macroeconomic fluctuations. *Journal of Development Economics*, 91(2):211–226.

Francis, N. and Restrepo-Ángel, S. (2018). Sectoral and aggregate response to oil price shocks in the colombian economy: Svar and local projections approach. *Borradores de Economía*, (Borradores de Economía; No. 1055).

Gazzani, A., Venditti, F., and Veronese, G. (2024). Oil price shocks in real time. *Journal of Monetary Economics*, 144:103547.

Gertler, M., Gilchrist, S., and Natalucci, F. M. (2007). External constraints on monetary policy and the financial accelerator. *Journal of Money, Credit and Banking*, 39(2-3):295–330.

Goda, T. and García, A. T. (2015). Flujos de capital, recursos naturales y enfermedad holandesa: el caso colombiano. *Ensayos sobre política económica*, 33(78):197–206.

Gomes, D., Kolovich, L., and Yi Wei, H. (2025). Oil Shocks and Labor Market Developments. *IMF Working Papers*, 2025(145):1.

Gómez, A. F. O. and Suárez, L. P. S. (2019). The importance of terms of trade in the colombian economy. *Monetary regimes and labour institutions: an alternative interpretation of the downward trend in exchange-rate pass-through in peripheral countries*, page 113.

González, A., López, M., Rodríguez, N., and Téllez, S. (2014). Fiscal policy in a small open economy with oil sector and non-ricardian agents. *Desarrollo y Sociedad*, (73):33–69.

González, S. and Hernández, E. (2016). Impactos indirectos de los precios del petróleo en el crecimiento económico colombiano. *Lecturas de Economía*, (84):113–141.

Greenwood, J., Hercowitz, Z., and Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. *The American Economic Review*, pages 402–417.

Guajardo, J. (2008). Business Cycles in Small Developed Economies. *IMF Working Papers*, 2008(086):1.

Hamann-Salcedo, F. A., Bejarano, J., and Rodríguez-Guzmán, D. A. (2015). Monetary policy implications for an oil-exporting economy of lower long-run international oil prices. *Borradores de Economía*, (Borradores de Economía; No. 871).

Hamilton, J. D. (1983). Oil and the macroeconomy since world war ii. *Journal of political economy*, 91(2):228–248.

Hernandez Jimenez, G. (2013). Terms of trade and output fluctuations in colombia. *Revista CEPAL*.

Horvath, J. and Yang, G. (2022). Unemployment dynamics and informality in small open economies. *European Economic Review*, 141:103991.

Jacho, D., Cruz, Z., and Carrillo-Maldonado, P. (2024). Effect of terms of trade on the latin american labor market. *International Economics*, 180:100552.

Jiménez, J. R. P. (2010). El impacto de los precios del petróleo sobre el crecimiento económico en colombia. *Revista de economía del Rosario*, 13(1):75–116.

Känzig, D. R. (2021). The macroeconomic effects of oil supply news: Evidence from opec announcements. *American Economic Review*, 111(4):1092–1125.

Kehoe, T. J. and Ruhl, K. J. (2008). Are shocks to the terms of trade shocks to productivity? *Review of Economic Dynamics*, 11(4):804–819.

Kilian, L. (2009). Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market. *American economic review*, 99(3):1053–1069.

Kilian, L. and Murphy, D. P. (2014). The role of inventories and speculative trading in the global market for crude oil. *Journal of Applied econometrics*, 29(3):454–478.

Kilian, L., Rebucci, A., and Spatafora, N. (2009). Oil shocks and external balances. *Journal of international Economics*, 77(2):181–194.

Koop, G. and Korobilis, D. (2010). Bayesian multivariate time series methods for empirical macroeconomics. *Foundations and trends in econometrics*, 3(4):267–358.

Kose, M. A. (2002). Explaining business cycles in small open economies:'how much do world prices matter?'. *Journal of International Economics*, 56(2):299–327.

Lambert, F., Pescatori, A., and Toscani, F. (2020). Labor Market Informality and the Business Cycle. *IMF Working Papers*, 2020(256):1.

Lenza, M. and Primiceri, G. E. (2022). How to estimate a vector autoregression after march 2020. *Journal of Applied Econometrics*, 37(4):688–699.

López-Piñeros, M. R. (2001). Seigniorage and the welfare cost of inflation in colombia. *Revista Ensayos Sobre Política Económica*, (Revista Ensayos Sobre Política Económica; Vol. 19. No. 39. Junio, 2001. Pág.: 115-131.).

Melo-Becerra, L. A., Parrado-Galvis, L. M., Ramos-Forero, J. E., and Zarate-Solano, H. M. (2020). Effects of booms and oil crisis on colombian economy: A time-varying vector autoregressive approach. *Revista de Economía del Rosario*, 23(1):31–63.

Mendoza, E. G. (1991). Real business cycles in a small open economy. *American Economic Review*, 81(4):797–818.

Mendoza, E. G. (1995). The terms of trade, the real exchange rate, and economic fluctuations. *International Economic Review*, 36(1):101–137.

Mertens, K. and Ravn, M. O. (2013). The dynamic effects of personal and corporate income tax changes in the united states. *American economic review*, 103(4):1212–1247.

Naraidoo, R. and Paez-Farrell, J. (2023). Commodity price shocks, labour market dynamics and monetary policy in small open economies. *Journal of Economic Dynamics and Control*, 151:104654.

Ojeda-Joya, J. N., Parra-Polanía, J. A., and Vargas, C. O. (2016). Fiscal rules as a response to commodity shocks: A welfare analysis of the colombian scenario. *Economic Modelling*, 52:859–866.

Ordóñez, J., Sala, H., and Silva, J. I. (2011). Oil price shocks and labor market fluctuations. *The Energy Journal*, pages 89–118.

Otero, J. D. Q. (2020). Not all sectors are alike: Differential impacts of shocks in oil prices on the sectors of the colombian economy. *Energy economics*, 86:104691.

Peersman, G. and Van Robays, I. (2012). Cross-country differences in the effects of oil shocks. *Energy Economics*, 34(5):1532–1547.

Perry, G. (2010). Hacia una reforma tributaria estructural. en r. steiner, & lv traverso. *Colombia: 2010-2014, Propuestas de Política Pública.*

Poncela, P., Senra, E., and Sierra, L. P. (2017). Long-term links between raw materials prices, real exchange rate and relative de-industrialization in a commodity-dependent economy: empirical evidence of “dutch disease” in colombia. *Empirical Economics*, 52(2):777–798.

Prada-Sarmiento, J. D. and Rojas, L. E. (2009). La elasticidad de frisch y la transmisión de la política monetaria en colombia. *Borradores de Economía; No. 555.*

Quero-Virla, L. (2016). Macroeconomic effects of oil price fluctuations in colombia. *Ecos de Economía*, 20(43):23–38.

Qureshi, I. A. and Ahmad, G. (2025). Oil price shocks and us business cycles. *Journal of Economic Dynamics and Control*, 177:105132.

Romer, C. D. and Romer, D. H. (2004). A new measure of monetary shocks: Derivation and implications. *American economic review*, 94(4):1055–1084.

Schmitt-Grohé, S. and Uribe, M. (2018). How important are terms-of-trade shocks? *International Economic Review*, 59(1):85–111.

Stock, J. H. and Watson, M. W. (2012). Disentangling the channels of the 2007-2009 recession. Technical report, National Bureau of Economic Research.

Stock, J. H. and Watson, M. W. (2018). Identification and estimation of dynamic causal effects in macroeconomics using external instruments. *The Economic Journal*, 128(610):917–948.

Toro, J., Garavito, A., López, D. C., Montes, E., et al. (2015). El choque petrolero y sus implicaciones en la economía colombiana. *Borradores de economía*, 906:1–65.

A Complete model

Variables: $c^H, c^L, C, k^{H,j}, i^{H,j}, K^j, I^j, h^{H,j}, h^{F,j}, h^{L,n}, H^{H,j}, H^{f,j}, H^{L,n}, b^{H,*}, B^*$, $\tilde{w}^{H,j}, \tilde{w}^{F,j}, \tilde{w}^{I,n}, R^{K,j}, p^\tau, p^n, p^x, p^m, q^j, y^j, m, x, H^L, T, \lambda^H, \lambda^L, tot, A^j$

$$k_t^{H,j} = (1-\delta)k_{t-1}^{H,j} + i_t^{H,j} \tag{1}$$

$$r_t^* = r_{ss}^* - \Phi_B \left(\exp(B_t^* - B_{ss}^*) - 1\right) \tag{2}$$

$$(c_t^H)^{-\sigma} = \lambda_t^H \tag{3}$$

$$\psi_H (h_t^{H,j})^{\eta_H} = \lambda_t^H \tilde{w}_t^{H,j} \tag{4}$$

$$p_t^\tau \lambda_t^H = \beta \mathbf{E}_t \lambda_{t+1}^H p_{t+1}^\tau (1 + r_t^*) \tag{5}$$

$$\lambda_t^H \left(1 + \Phi_j (k_t^{H,j} - k_{t-1}^{H,j})\right) = \beta \mathbf{E}_t \lambda_{t+1}^H \left(R_{t+1}^{H,j} + 1 - \delta + \Phi_j (k_{t+1}^{H,j} - k_t^{H,j})\right) \tag{6}$$

$$c_t^L = \sum_{j=m,x,n} \tilde{w}_t^{F,j} h_t^{F,j} + \tilde{w}_t^{I,n} h_t^{I,n} + T_t \tag{7}$$

$$(c_t^L)^{-\sigma} = \lambda_t^L \tag{8}$$

$$\psi_F (h_t^{F,j})^{\eta_L} = \lambda_t^L \tilde{w}_t^{F,j} \tag{9}$$

$$\psi_I (h_t^{I,j})^{\eta_L} = \lambda_t^L \tilde{w}_t^{I,n} \tag{10}$$

$$C_t = N c_t^H + (1-N) c_t^L \tag{11}$$

$$B_t^* = N b_t^{H,*} \tag{12}$$

$$I_t^j = N i_t^{H,j} \tag{13}$$

$$K_t^j = N k_t^{H,j} \tag{14}$$

$$H_t^{H,j} = N h_t^{H,j} \tag{15}$$

$$H_t^{F,j} = (1-N) h_t^{F,j} \tag{16}$$

$$H_t^{I,n} = (1-N) h_t^{I,n} \tag{17}$$

$$q_t^\tau = \omega_c (p_t^\tau)^{-\theta_c} \left(C_t + \sum_{j=m,x,n} I_t^j \right) \tag{18}$$

$$q_t^n = (1-\omega_c)(p_t^n)^{-\theta_c} \left(C_t + \sum_{j=m,x,n} I_t^j \right) \tag{19}$$

$$1 = \left[\omega_c (p_t^\tau)^{1-\theta_c} + (1-\omega_c)(p_t^n)^{1-\theta_c} \right]^{\frac{1}{1-\theta_c}} \tag{20}$$

$$q_t^x = \omega_\tau \left(\frac{p_t^x}{p_t^\tau} \right)^{-\theta_\tau} q_t^\tau \tag{21}$$

$$q_t^m = (1-\omega_\tau) \left(\frac{p_t^m}{p_t^\tau} \right)^{-\theta_\tau} q_t^\tau \tag{22}$$

$$p_t^\tau = \left[\omega_\tau (p_t^x)^{1-\theta_\tau} + (1-\omega_\tau)(p_t^m)^{1-\theta_\tau} \right]^{\frac{1}{1-\theta_\tau}} \tag{23}$$

$$y_t^x = A_t^x (K_t^x)^{\alpha_x} (H_t^{H,x})^{\kappa_x} (H_t^{F,x})^{1-\alpha_x-\kappa_x} \tag{24}$$

$$\alpha_x \frac{p_t^x y_t^x}{K_t^x} = R_t^x \tag{25}$$

$$\kappa_x \frac{p_t^x y_t^x}{H_t^{H,x}} = (1+\tau^H) \tilde{w}_t^{H,x} \tag{26}$$

$$(1-\alpha_x-\kappa_x) \frac{p_t^x y_t^x}{H_t^{F,x}} = (1+\tau^F) \tilde{w}_t^{F,x} \tag{27}$$

$$x_t = p_t^x (y_t^x - q_t^x) \tag{28}$$

$$y_t^m = A_t^m (K_t^m)^{\alpha_x} (H_t^{H,m})^{\kappa_m} (H_t^{F,m})^{1-\alpha_m-\kappa_m} \tag{29}$$

$$\alpha_m \frac{p_t^m y_t^m}{K_t^m} = R_t^m \tag{30}$$

$$\kappa_m \frac{p_t^m y_t^m}{H_t^{H,m}} = (1+\tau^H) \tilde{w}_t^{H,m} \tag{31}$$

$$(1-\alpha_m-\kappa_m) \frac{p_t^m y_t^m}{H_t^{F,m}} = (1+\tau^F) \tilde{w}_t^{F,m} \tag{32}$$

$$m_t = p_t^m (q_t^m - y_t^m) \tag{33}$$

$$y_t^n = A_t^n (K_t^n)^{\alpha_x} (H_t^{H,n})^{\kappa_m} (H_t^{L,n})^{1-\alpha_n-\kappa_n} \tag{34}$$

$$H_t^{L,n} = \left[\omega_L^{\frac{1}{\theta_L}} (H_t^{F,n})^{\frac{\theta_L-1}{\theta_L}} + (1-\omega_L)^{\frac{1}{\theta_L}} (H_t^{I,n})^{\frac{\theta_L-1}{\theta_L}} \right]^{\frac{\theta_L}{\theta_L-1}} \tag{35}$$

$$\alpha_n \frac{p_t^n y_t^n}{K_t^n} = R_t^n \tag{36}$$

$$\kappa_n \frac{p_t^n y_t^n}{H_t^{H,n}} = (1+\tau^H)\tilde{w}_t^{H,n} \tag{37}$$

$$(1-\alpha_n-\kappa_n)\frac{p_t^n y_t^n}{H_t^{L,n}}\omega_L^{\frac{1}{\theta_L}}\left(\frac{H_t^{L,n}}{H_t^{F,n}}\right)^{\frac{1}{\theta_L}} = (1+\tau^F)\tilde{w}_t^{F,n} \tag{38}$$

$$(1-\alpha_n-\kappa_n)\frac{p_t^n y_t^n}{H_t^{L,n}}(1-\omega_L)^{\frac{1}{\theta_L}}\left(\frac{H_t^{L,n}}{H_t^{I,n}}\right)^{\frac{1}{\theta_L}} = \tilde{w}_t^{I,n} \tag{39}$$

$$y_t^n = q_t^n \tag{40}$$

$$T_t = \tau^H \sum_{j\in x,m,n} H_t^{H,j} + \tau^F \sum_{j\in x,m,n} H_t^{F,j} \tag{41}$$

$$p_t^\tau B_t^* - p_t^\tau(1+r_{t-1}^*)B_{t-1}^* = x_t - m_t \tag{42}$$

$$tot_t = \frac{p_t^x}{p_t^m} \tag{43}$$

$$tot_t = (tot_{t-1})^{\rho_{tot}}(tot_{ss})^{1-\rho_{tot}}(1+\varepsilon_t^{tot}) \tag{44}$$

$$A_t^j = (A_{t-1}^j)^{\rho_{aj}}(A_{ss}^j)^{1-\rho_{aj}}(1+\varepsilon_t^{aj}) \tag{45}$$

B Welfare

Welfare measurement. Household welfare is measured using a recursive value function defined over the equilibrium path following the shock. For each household $j \in \{H, L\}$, the value function satisfies

$$W_j(t) = u_j(t) + \beta\, W_j(t+1), \tag{1}$$

where $u_j(t)$ is the period utility flow. Under GHH preferences, the utility flows are

$$u_H(t) = \frac{1}{1-\sigma}\left(c_t^H - \frac{1}{\psi_H}\sum_{s\in\{x,m,n\}}\left(h_t^{H,s}\right)^{1+\eta_H}\right)^{1-\sigma}, \tag{2}$$

$$u_L(t) = \frac{1}{1-\sigma}\left(c_t^L - \frac{1}{\psi_F}\sum_{s\in\{x,m,n\}}\left(h_t^{F,s}\right)^{1+\eta_L} - \frac{1}{\psi_I}\left(h_t^{I,n}\right)^{1+\eta_L}\right)^{1-\sigma}, \tag{3}$$

whereas under separable preferences they take the form

$$u_H(t) = \frac{(c_t^H)^{1-\sigma}}{1-\sigma} - \frac{1}{\psi_H(1+\eta_H)}\sum_{s\in\{x,m,n\}}\left(h_t^{H,s}\right)^{1+\eta_H}, \tag{4}$$

$$u_L(t) = \frac{(c_t^L)^{1-\sigma}}{1-\sigma} - \frac{1}{\psi_F(1+\eta_L)}\sum_{s\in\{x,m,n\}}\left(h_t^{F,s}\right)^{1+\eta_L} - \frac{1}{\psi_I(1+\eta_L)}\left(h_t^{I,n}\right)^{1+\eta_L}. \tag{5}$$

Equation (1) is included as an endogenous variable in the Dynare model, so the impulse response of $W_j(t)$ is solved jointly with the rest of the equilibrium system. The value at $t = 1$, $W_j^{\text{shock}}(1)$, already embed the full discounted sum of future utility flows along the transition path and therefore constitutes a sufficient statistic for the welfare impact of the shock. In the no-shock steady state, (1) reduces to

$$W_j^{ss} = \frac{u_j^{ss}}{1-\beta}, \qquad j \in \{H, L\}. \tag{6}$$

The welfare impact is expressed as a consumption-equivalent variation (CEV), defined as the permanent fraction ξ_j of steady-state consumption that would leave household j indifferent between remaining at the undisturbed steady state and experiencing the equilibrium path induced by the shock:

$$W_j^{ss}(1+\xi_j) = W_j^{\text{shock}}(1). \tag{7}$$

Solving (7) yields

$$\xi_j = \left(\frac{W_j^{\text{shock}}(1)}{W_j^{ss}}\right)^{\frac{1}{1-\sigma}} - 1. \tag{8}$$

A positive value of ξ_j indicates a welfare gain. Since $W_j^{ss} < 0$ for $\sigma > 1$, a welfare improvement implies $W_j^{\text{shock}}(1) > W_j^{ss}$ (i.e., less negative), so that the ratio in (8) is less than one in absolute value and $\xi_j > 0$.

Welfare effects. Table 3 reports the CEV for each household type and preference specification. In both cases the shock generates welfare gains, but the magnitude depends critically on the assumed preferences. Under GHH preferences, $\xi_H = 0.18$ % and $\xi_L = 0.26$ %, roughly three times larger than the corresponding figures under separable preferences ($\xi_H = 0.06$ and $\xi_L = 0.08$ %). This amplification reflects the larger consumption response documented in Figure 5: absent the wealth effect on labor supply, the income transfer from the terms-of-trade improvement translates directly into higher consumption for both households.

In both specifications, the low-skill household obtains larger welfare gains than the high-skill household ($\xi_L > \xi_H$). This distributional result is more pronounced under GHH preferences, where the gap $\xi_L - \xi_H = 0.08$ %age points, compared to 0.02 percentage points under separable preferences. The asymmetry reflects the additional margin available to low-skill workers: the expansion of the non-tradable sector under GHH increases both formal employment ($H_t^{F,n}$) and informal employment ($H_t^{I,n}$) for household L, generating wage and hours gains that compound the consumption effect. These results suggest that, while the aggregate welfare impact of a commodity price boom is modest, it carries a mildly pro-poor distributional profile, particularly when preferences eliminate the offsetting wealth effect on labor supply.

Table 3: Welfare effects of a terms-of-trade shock (CEV, %)

	GHH	Separable
Household H	0.1814	0.0570
Household L	0.2637	0.0765

Notes: Consumption-equivalent variation (CEV) defined as $\xi_j = \left(W_j^{\text{shock}}(1)/W_j^{ss}\right)^{1/(1-\sigma)} - 1$, where $W_j^{ss} = u_j^{ss}/(1-\beta)$ is the steady-state welfare level and $W_j^{\text{shock}}(1)$ is its value at $t = 1$ following a one-standard-deviation positive terms-of-trade shock. Both runs use rigid wages ($\phi_j > 0$) and GHH preferences in the baseline. $\sigma = 2$.